

# Cascades

Social and Technological Networks

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# Network cascades

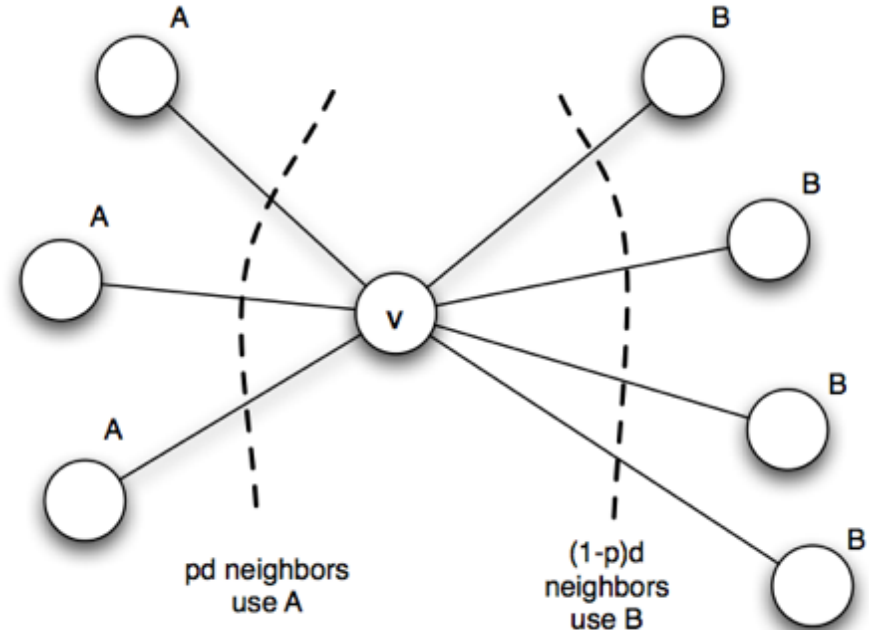
- Things that spread (diffuse) along network edges
- Epidemics
- Ideas
- Innovation:
  - We use technology our friends/colleagues use
  - Compatibility
  - Information/Recommendation/endorsement

# Models

- Basic idea: Your benefits of adopting a new behavior increases as more of your friends adopt it
- Technology, beliefs, ideas... a “contagion”

# Contagion Threshold

- $v$  has  $d$  edges
- $p$  fraction use A
- $(1-p)$  use B
- $v$ 's benefit in using A is  $a$  per A-edge
- $v$ 's benefit in using B is  $b$  per B-edge



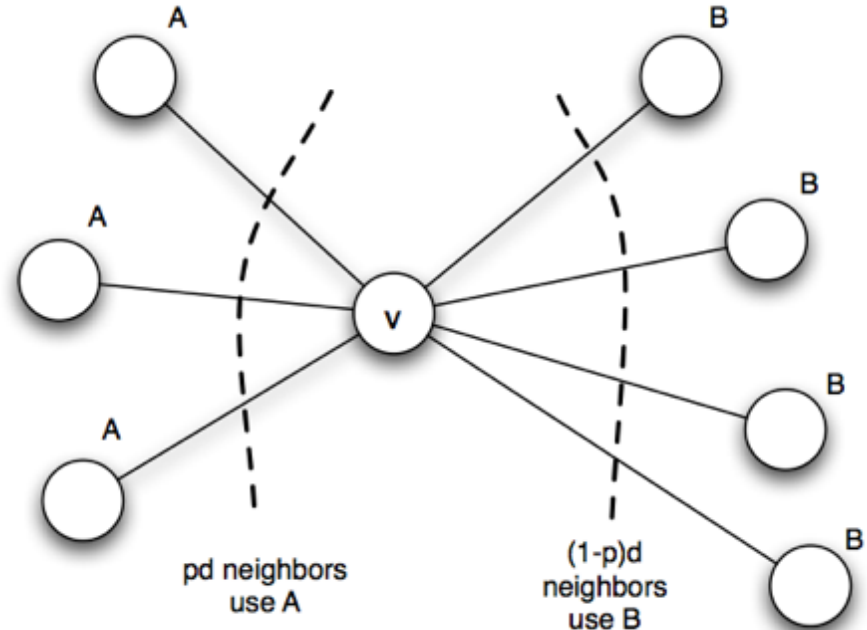
# Contagion Threshold

- A is a better choice if:

$$pda \geq (1 - p)db,$$

- or:

$$p \geq \frac{b}{a + b}.$$



# The contagion threshold

- Let us write  $q = b/(a+b)$
- If  $q$  is small, that means  $b$  is small relative to  $a$ 
  - Therefore  $A$  is useful even if only a small fraction of neighbors are using it
- If  $q$  is large, that means the opposite is true, and  $B$  is a better choice

# Cascading behavior

- If everyone is using A (or everyone is using B)
- There is no reason to change — equilibrium
- If both are used by some people, the network state may change towards one or the other.
  - Cascades: We want to understand how likely that is.
- Or there may be a deadlock
  - Equilibrium: We want to understand what that may look like

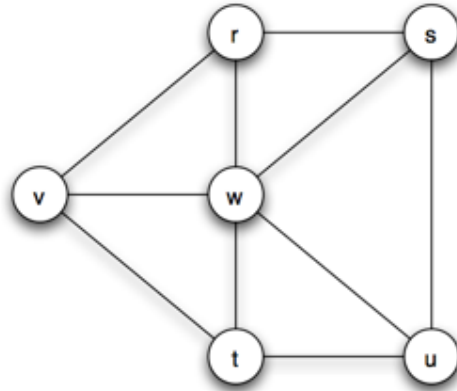
# Cascades

- Suppose initially everyone uses B
- Then some small number adopts A
  - For some reason outside our knowledge
- Will the entire network adopt A?
- What will cause A's spread to stop?

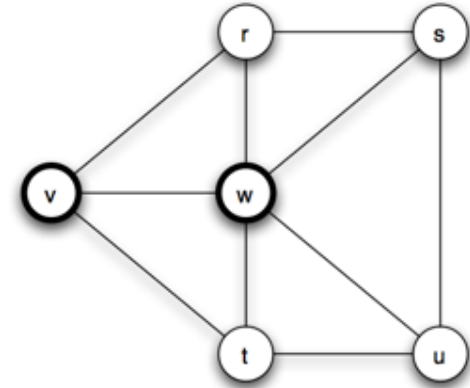


# Example

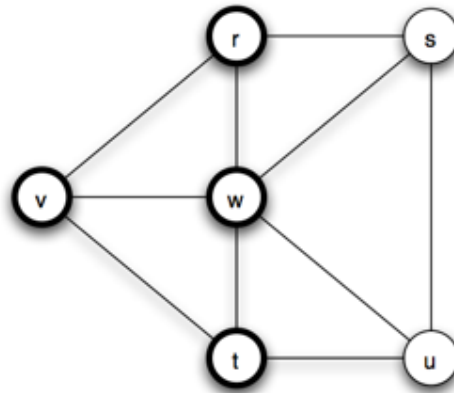
- $a = 3, b = 2$
- $q = 2/5$



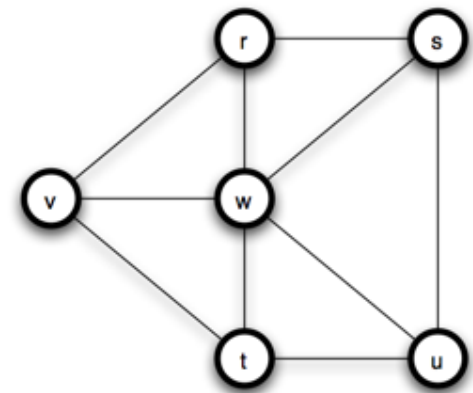
(a) *The underlying network*



(b) *Two nodes are the initial adopters*



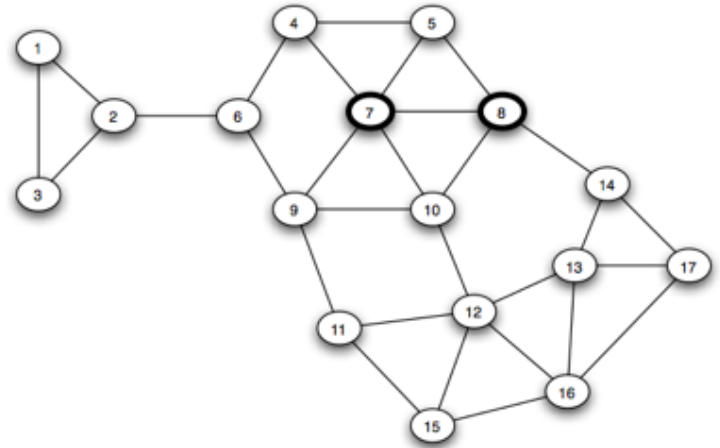
(c) *After one step, two more nodes have adopted*



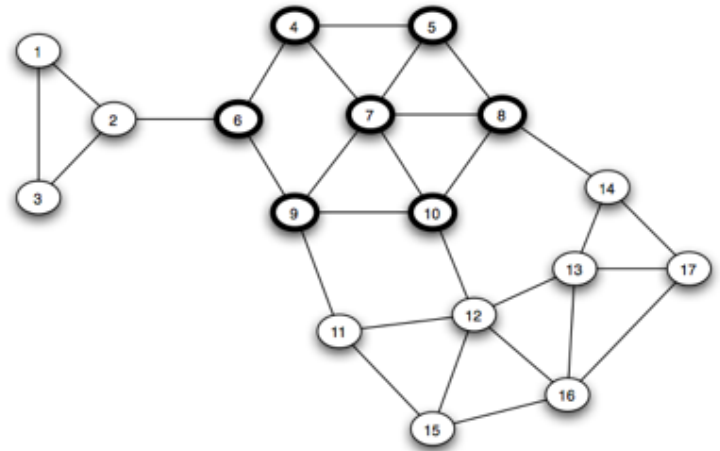
(d) *After a second step, everyone has adopted*

# Example

- $a = 3, b = 2$
- $q = 2/5$



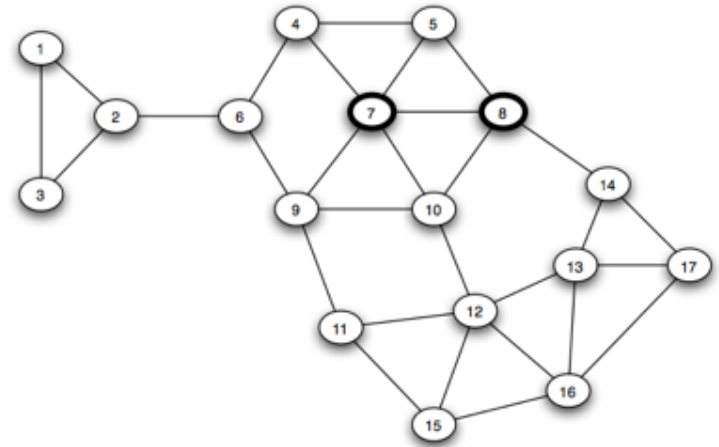
(a) *Two nodes are the initial adopters*



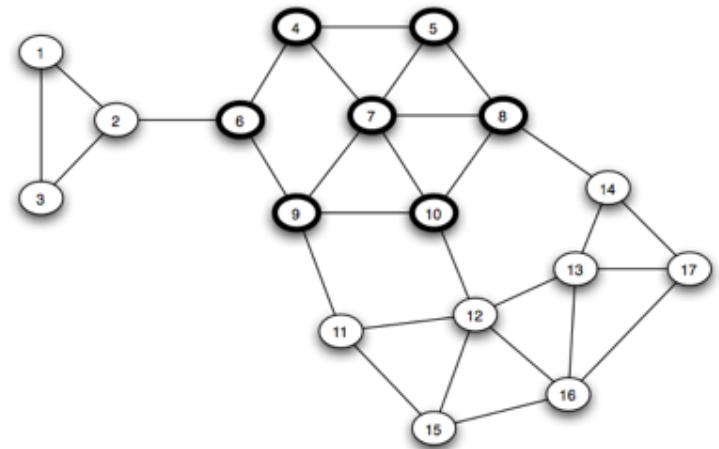
(b) *The process ends after three steps*

# Spreading innovation

- A can be made to spread more by making a better product,
- say  $a = 4$ , then  $q = 1/3$
- and A spreads
- Or, convince some key people to adopt A
- node 12 or 13



(a) Two nodes are the initial adopters



(b) The process ends after three steps

# Stopping of spread

- Tightly knit communities stop the spread
  - More easily for “complex contagion” that need multiple enforcements
- Weak links are good for information transmission, not for behavior transmission
- Political conversion is rare
- Certain social networks are popular in certain demographics
- You can defend your “product” by creating tight communities among users

# $\alpha$ - strong communities

- Let us write  $d_S(v)$  for the degree of  $v$  in a subset of nodes  $S$
- The set  $S$  of nodes forms an  $\alpha$ -strong (or  $\alpha$ -dense) community if for each node  $v$  in  $S$ ,  
 $d_S(v) \geq \alpha d(v)$
- That is, at least  $\alpha$  fraction of neighbors of each node is within the community

# Theorem

- A cascade with contagion threshold  $q$  cannot penetrate an  $\alpha$ -dense community with  $\alpha > 1 - q$
- Therefore, for a cascade with threshold  $q$ , and set  $X$  of initial adopters of  $A$ :
  1. If the rest of the network contains a cluster of density  $> 1 - q$ , then the cascade from  $X$  does not result in a complete cascade
  2. If the cascade is not complete, then the rest of the network must contain a cluster of density  $> 1 - q$

# Proof

- In Kleinberg & Easley
- By contradiction: The first node in the cluster that converts, cannot convert.
- If set  $S$  is exactly the set of unconverted nodes at the end, then any  $v$  in  $S$  must have  $1-q$  fraction edges in  $S$ , else  $v$  would have converted.

# Extensions

- The model extends to the case where each node  $v$  has
  - different  $a_v$  and  $b_v$ , hence different  $q_v$
  - Exercise: What can be a form for the theorem on the previous slide for variable  $q_v$ ?

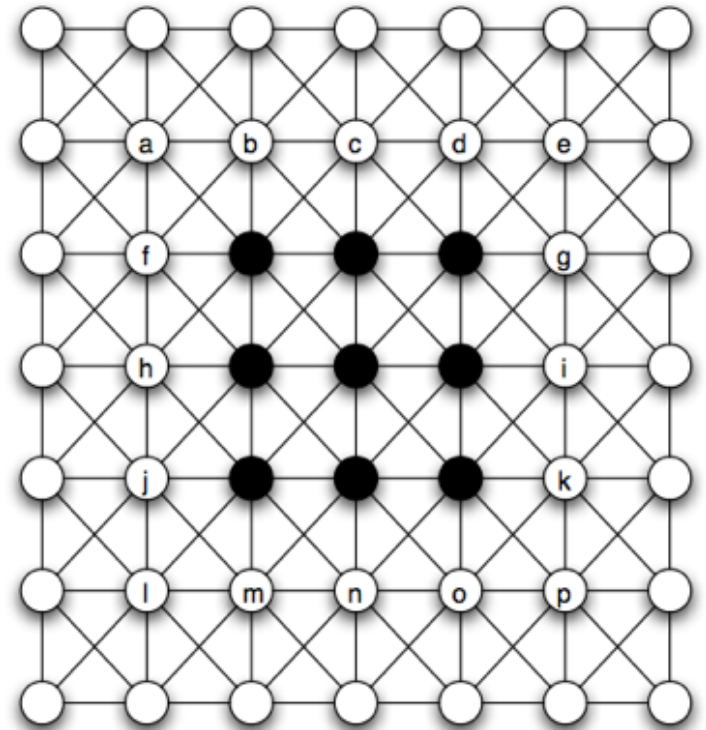


# Cascade capacity

- Upto what threshold  $q$  can a small set of early adopters cause a full cascade?
- definition: Small: A finite set in an infinite network

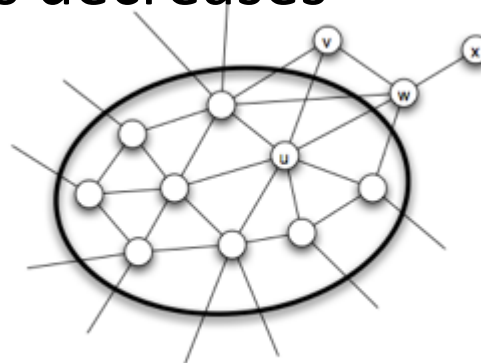
# Cascade capacities

- 1-D grid:
- capacity =  $1/2$
  
- 2-D grid with 8 neighbors:
- capacity  $3/8$



# Theorem

- No infinite network has cascade capacity  $> 1/2$
- Show that the interface/boundary shrinks
- Number of edges at boundary decreases at every step
- Take a node  $w$  at the boundary that converts in this step
- $w$  had  $x$  edges to  $A$ ,  $y$  edges to  $B$
- $q > 1/2$  implies  $x > y$
- True for all nodes
- Implies boundary edges decreases



(a) Before  $v$  and  $w$  adopt  $A$



(b) After  $v$  and  $w$  adopt  $A$

- Implies, an inferior technology cannot win an infinite network
- Or: In a large network inferior technology cannot win with small starting resources

# Other models

- Non-monotone: an infected/converted node can become un-converted
- Schelling's model, granovetter's model:  
People are aware of choices of all other nodes  
(not just neighbors)

# Causing large spread of cascade

- Viral marketing with restricted costs
- Suppose you have a budget of reaching  $k$  nodes
- Which  $k$  nodes should you convert to get as large a cascade as possible?

# Models

- Linear contagion threshold model:
- The model we have used: node activates to use A if benefit of using  $p > q$
- Independent activation model:
- If node  $u$  activates to use A, then  $u$  causes neighbor  $v$  to activate and use A with probability  
–  $p_{u,v}$
- That is, every edge has an associated probability of spreading influence (like the strength of the tie)

# Hardness

- In both the models, finding the exact set of  $k$  initial nodes to maximize the influence cascade is NP-Hard
  - Intractable, unlikely that polynomial time algorithms exist unless  $P = NP$



# Project submission

- Instructions online (see link to projects)
- Link to sample projects from Stanford course
- Ask questions on piazza
- We understand that this is a short course project. Perfect reports are not possible
- Try to show us something(s) interesting, promising
- Try unusual ideas. Explain them & results cleanly
- Use your imagination, have fun!