There are two ways of writing the power law (see Kempe 2011). Suppose $X$ is the variable in question, then:

- $\Pr[X \geq x] = cx^{-\alpha}$
- $\Pr[X = x] = c'x^{-\alpha'}$

**Exercise 0.1.** Express $(c', \alpha')$ in terms of $(c, \alpha)$ and vice versa.

**Exercise 0.2.** Consider the preferential attachment model discussed in class. In what ways do you think it is different from real power law networks like the www and others?

**Exercise 0.3.** It is the in-degrees of nodes in www that are expected of have power law distribution. The code we tried in class took all degrees. Write your version to plot the in-degrees.

* **Exercise 0.4.** Show that preferential attachment networks have small diameter. (Take all edges as undirected.)

* **Exercise 0.5.** Do preferential attachment networks have expansion above a constant? (Take all edges as undirected.)

**Exercise 0.6.** If the number of neighbors $k$ is a constant in a Watts-Strogatz graph, (whereas $n$ is not a constant – the graph can be very large), and the fraction of rewired edges $\beta$ is also a constant, show that there exists a constant $c$ such that for any vertex $u$, the expected clustering coefficient is greater than $c$.

**Exercise 0.7.** In Kleinberg’s model, suppose instead of a 2D grid, we started with a 3D grid. What do you think is the distribution needed for decentralized search to work with the same $\log^2 n$ guarantee?

**Exercise 0.8.** In an $n \times n$ grid, check that the number of nodes at graph distance exactly $d$ from any node is $\Theta(d)$, and number of nodes in the ball $B_d(x)$ is $\Theta(d^2)$. What is the shape of this ball?

**Exercise 0.9.** Suppose that a long link from node $u$ will connect to node $v$ with probability proportional to $\frac{1}{d^2}$ where $d$ is the grid distance between $u$ and $v$. Show that:
1. The proportionality constant for any node is $\Theta(\log n)$. 

2. The probability that a long link from $u$ has its other end point in $B_{d/2}(v)$ is $\frac{1}{\Theta(\log n)}$. 

Exercise 0.10. Do the properties hold if the base graph is a balanced binary tree?