

## Lecture 4. Power law networks

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Class notes

We covered the following in class:

- www shows a power law degree distribution
- The exponential vs polynomial drop makes a large difference: Number of hubs.
- Log-log plots are useful in seeing power law, but should be used with care
- The mean of power law is finite iff  $\alpha > 2$ .
- Preferential attachment mechanism and model
- power law can arise from other types of optimization and heuristics

There are two ways of writing the power law (see [Kempe, 2011]). Suppose  $X$  is the variable in question, then:

- $\Pr[X \geq x] = cx^{-\alpha}$

OR

- $\Pr[X = x] = c'x^{-\alpha'}$

**Exercise 0.1.** Express  $(c', \alpha')$  in terms of  $(c, \alpha)$  and vice versa.

\* **Exercise 0.2.** Show that preferential attachment networks have small diameter. (Take all edges as undirected.)

\* **Exercise 0.3.** Do preferential attachment networks have expansion above a constant? (Take all edges as undirected.)

\* **Exercise 0.4.** What do you think happens to diameter and expansion in real world power law networks?

**Exercise 0.5.** It is the in-degrees of nodes in www that are expected to have power law distribution. The code we tried in class took all degrees. Write your version to plot the in-degrees.

## References

[Kempe, 2011] Kempe, D. (2011). Structure and dynamics of information in networks, lecture notes. Technical report, U.S.C.