Structure and Synthesis of Robot Motion

Motion Synthesis in Groups and Formations I

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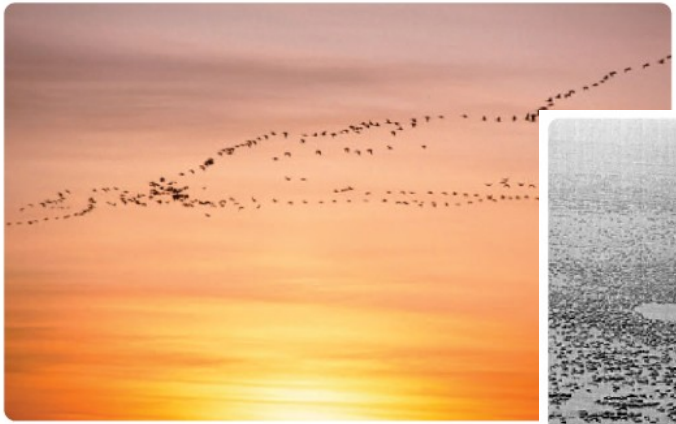
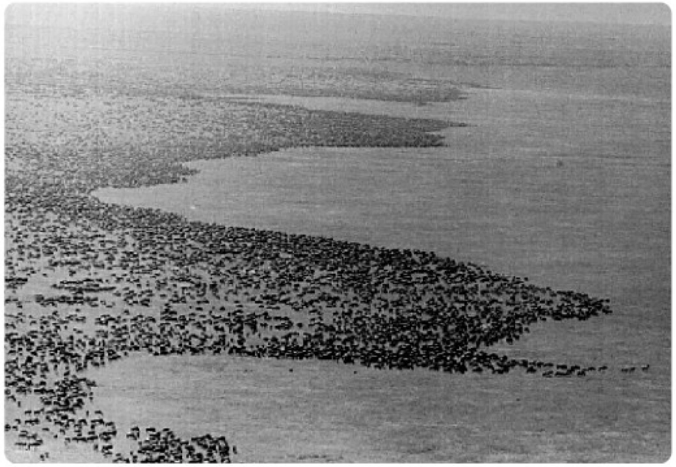
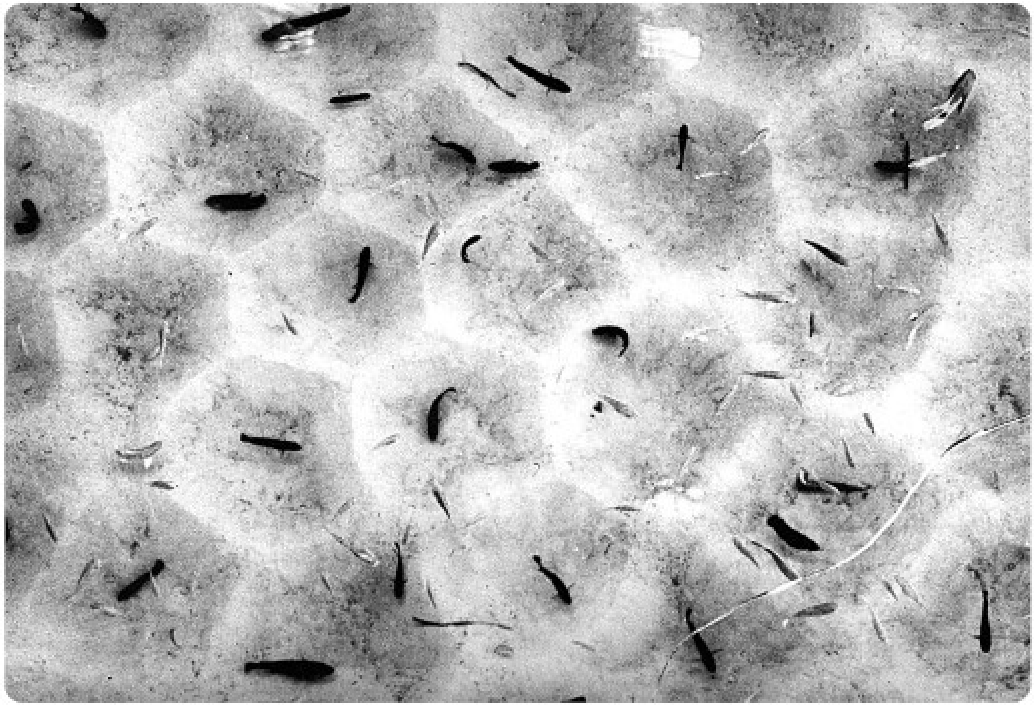
Consider Motion Problems with Many Agents





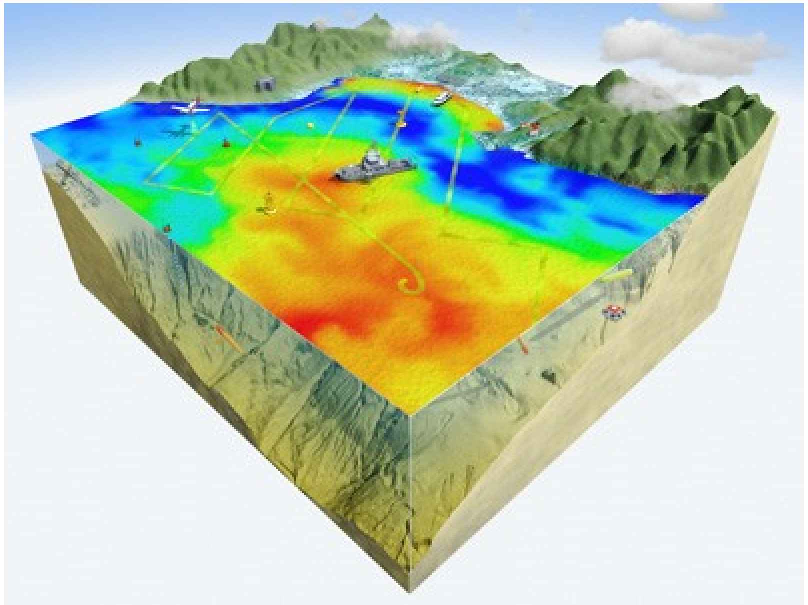
How should we model and solve these problems?

Interesting Effects in Such Groups



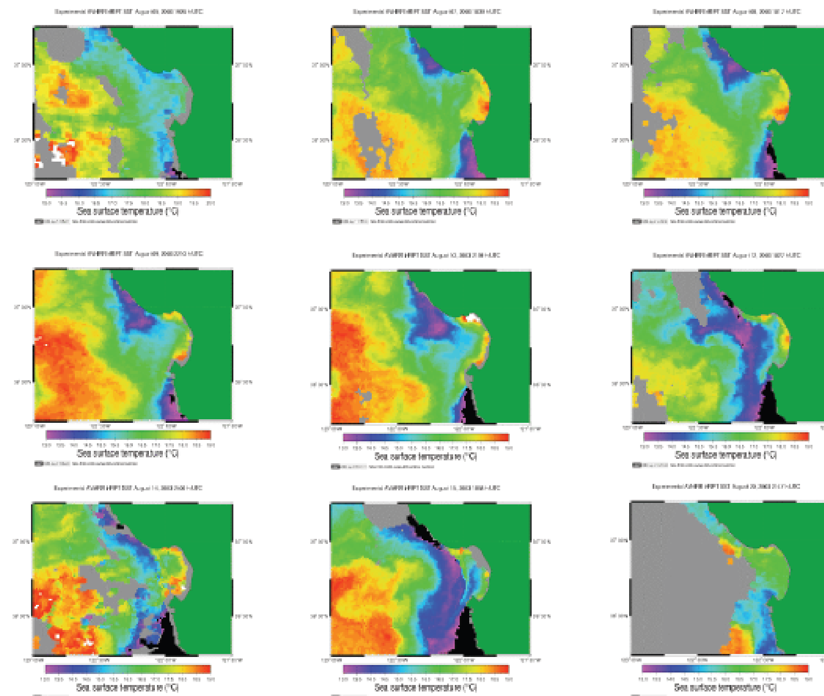
Territorial behaviour in tilapia mossambica
- No single fish has this global view so what do they do?

Increasingly Important within Robotics



N.E. Leonard et al., Collective Motion, Sensor Networks, and Ocean Sampling, *Proc. IEEE* 95(1):48–74, 2007.

The Autonomous Ocean Sampling Network

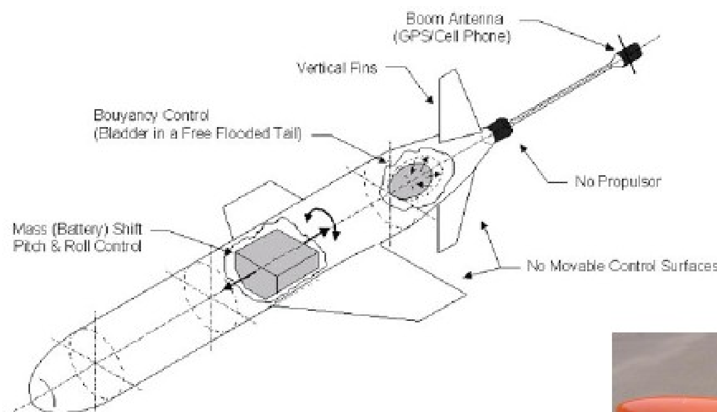


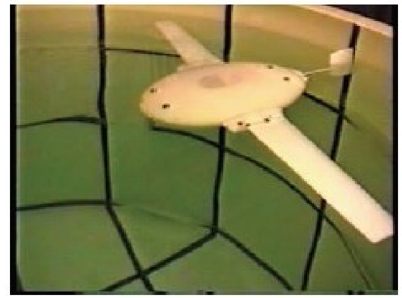
Satellite Sea Surface Temperature (SST),
Monterey Bay, CA, Aug 5-20, 2003

The Robots

Buoyancy Driven Gliders

- Autonomous vehicles with very long range (i.e., lifetime)





ROGUE

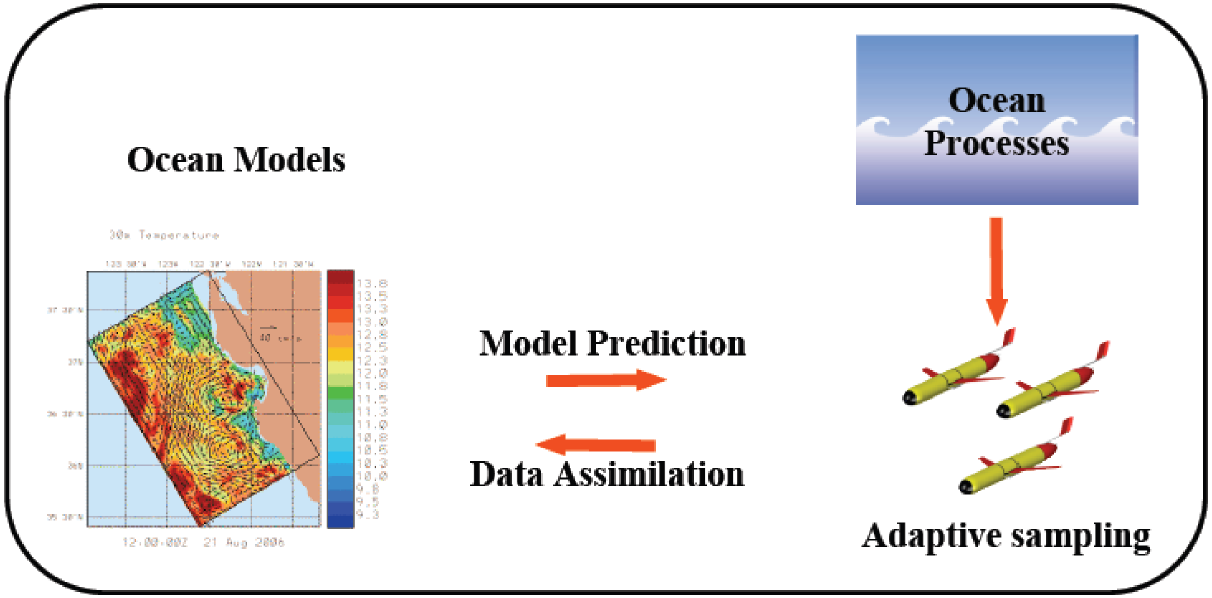
Slocum Glider

Spray Glider

Scientific Goals

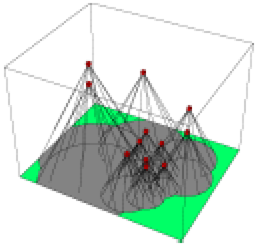
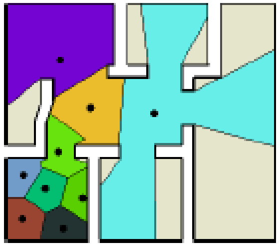
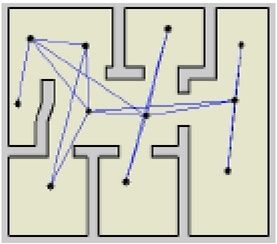
Sustainable, portable, adaptive, coupled observation/modeling system.

Learn how to deploy, direct and utilize autonomous vehicles most efficiently to sample the ocean, assimilate the data into numerical models in real time, and predict future conditions with minimal error.



Cooperative Multi-agent Systems

- What kind of systems?
  - Groups of agents with control, sensing, communication and computing
- Each individual
  - senses its immediate environment
  - communicates with others
  - processes information gathered
  - takes local action in response

Motivation: Decision Making in Animals

- Able to
  - deploy over a given region
  - assume specified pattern
  - rendezvous at a common point
  - jointly initiate motion/change direction in a synchronized way

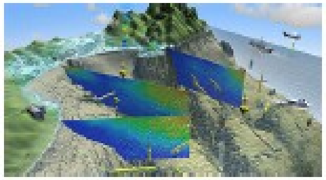
- Species achieve synchronized behavior
  - with limited sensing/communication between individuals
  - without apparently following group leader

(Couzin et al, Nature 05; Conradt et al, Nature 03)

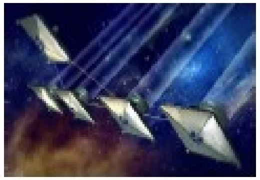
Motivation: Engineered Multi-agent Systems

- Embedded robotic systems and sensor networks for
  - high-stress, rapid deployment — e.g., disaster recovery networks
  - distributed environmental monitoring — e.g., portable chemical and biological sensor arrays detecting toxic pollutants
  - autonomous sampling for biological applications — e.g., monitoring of species in risk, validation of climate and oceanographic models
  - science imaging — e.g., multi-spacecraft distributed interferometers flying in formation to enable imaging at micro-arcsecond resolution

Sandia National Labs



MBARI AOSN



NASA Terrestrial Planet Finder

Challenges

- What useful engineering tasks can be performed with limited-sensing/communication agents?
  - Feedback: rather than open-loop computation for known/static setup
  - Information flow: who knows what, when, why, how, dynamically changing
  - Reliability/performance: robust, efficient, predictable behavior

- How to coordinate individual agents into coherent whole?

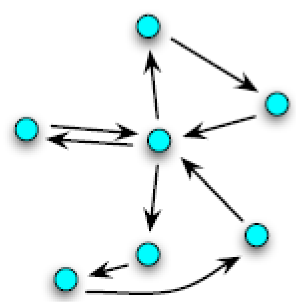
Objective: systematic methodologies to design and analyze cooperative strategies to control multi-agent systems

Integration of control, communication, sensing, computing

Notion of Distributed Algorithms

- Simplest distributed iteration is linear averaging:
  - you are given a graph
  - each node contains a value x_i
  - each node repeatedly executes:

$$x^+_i := average(x_i, \{x_j \text{ , for all neighboring } j\})$$



Why does this algorithm converge and to what?

Idea for Why Distributed Averaging Works

- If you define the pair-wise disagreement as a contributor to an 'energy'
- then the iterations act to minimize the energy

$$\Phi_G(x) = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij}(x_j - x_i)^2$$

- Many ways to understand such processes:
  - Matrix Analysis (Perron-Frobenius theory)
  - Algebraic Graph Theory (Laplacians and flows)

Recap: Potential Function

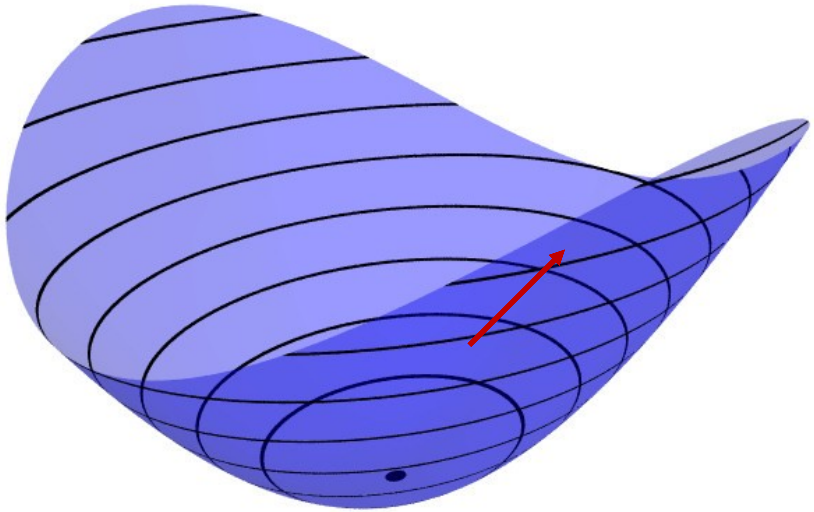
- Differentiable real-valued function,

$$U : \mathbb{R}^m \mapsto \mathbb{R}$$

- Treat the value as 'energy'
- Then, gradient is the vector,

$$\nabla U(q) = DU(q)' = [\frac{\partial U}{\partial q_1}(q), \ldots, \frac{\partial U}{\partial q_m}(q)]'$$

- The gradient points in the direction that locally maximally increases U

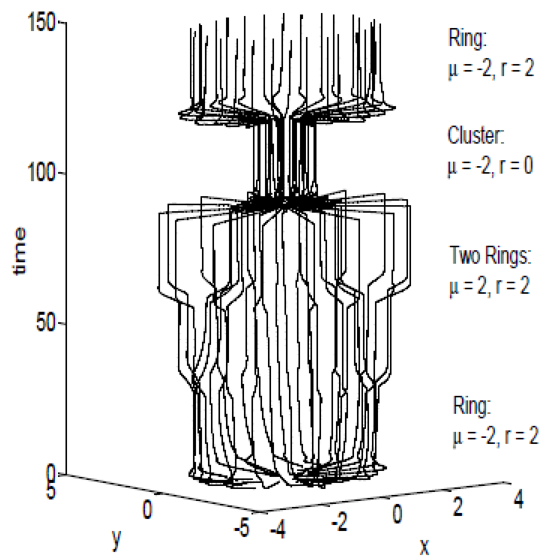


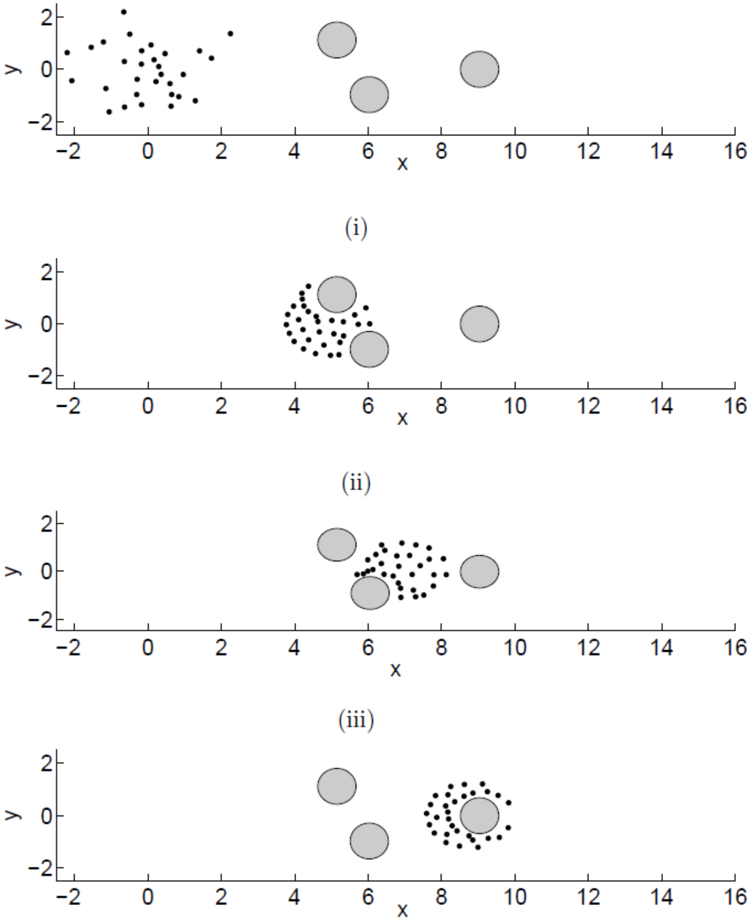
We could use such an encoding for the motion of the group. How exactly do we encode?

Potential for Steering to a 'Ring'

'Tune' formation on to a ring:

$$U^S(\mathbf{x}_i;\mu,\alpha) = -\frac{1}{2}\mu(\rho_i - r)^2 + \frac{1}{4}(\rho_i - r)^4 + \frac{1}{2}\alpha z_i^2$$





Behaviour of Ring Potential Function

- Define: $D = \frac{\partial^2 U}{\partial \rho_i^2}\frac{\partial^2 U}{\partial z_i^2} - \left[\frac{\partial^2 U}{\partial \rho_i \partial z_i}\right]^2$

- Conditions for stability:

(i) $D > 0$, $\partial^2 U/\partial \rho_i^2 > 0 \implies$ equilibrium point is a stable minimum.
(ii) $D > 0$, $\partial^2 U/\partial \rho_i^2 < 0 \implies$ equilibrium point is an unstable maximum.
(iii) $D < 0 \implies$ equilibrium point is a saddle.

$$\frac{\partial^2 U}{\partial \rho_i^2} = -\mu + 3(\rho_i - r)^2$$

$$\frac{\partial^2 U}{\partial z_i^2} = \alpha$$

$$\frac{\partial^2 U}{\partial \rho_i \partial z_i} = 0$$

Dynamics Under Steering Potential

- The induced equations of motion:

$$\begin{pmatrix} \dot{\mathbf{x}}_i \\ \dot{\mathbf{v}}_i \end{pmatrix} = \begin{pmatrix} \mathbf{v}_i \\ -\sigma \mathbf{v}_i - \nabla_i U^S(\mathbf{x}_i) \end{pmatrix} = \begin{pmatrix} f(\mathbf{x}_i, \mathbf{v}_i) \\ g(\mathbf{x}_i, \mathbf{v}_i) \end{pmatrix}$$

- And its local linearization:

$$\begin{pmatrix} \delta\dot{\mathbf{x}}_i \\ \delta\dot{\mathbf{v}}_i \end{pmatrix} = \mathbf{J} \begin{pmatrix} \delta\mathbf{x}_i \\ \delta\mathbf{v}_i \end{pmatrix}$$

where,

$$\mathbf{J} = \begin{pmatrix} \frac{\partial}{\partial \mathbf{x}_i}(f(\mathbf{x}_i, \mathbf{v}_i)) & \frac{\partial}{\partial \mathbf{v}_i}(f(\mathbf{x}_i, \mathbf{v}_i)) \\ \frac{\partial}{\partial \mathbf{x}_i}(g(\mathbf{x}_i, \mathbf{v}_i)) & \frac{\partial}{\partial \mathbf{v}_i}(g(\mathbf{x}_i, \mathbf{v}_i)) \end{pmatrix}\Bigg|_{\mathbf{x}_o, \mathbf{v}_o}$$

Analyze eigenvalues to ensure stability

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\partial^2 U}{\partial \rho_i^2} & -\frac{\partial^2 U}{\partial \rho_i \partial z_i} & -\sigma & 0 \\ -\frac{\partial^2 U}{\partial \rho_i \partial z_i} & -\frac{\partial^2 U}{\partial z_i^2} & 0 & -\sigma \end{pmatrix}\Bigg|_{\mathbf{x}_o, \mathbf{v}_o}$$

Dynamics Under Steering Potential

- To understand 'nonlinear' stability, consider a Lyapunov function based on total energy

$$L = \sum_i \left(\frac{1}{2} |\mathbf{v}_i|^2 + U^S(\mathbf{x}_i) \right)$$

- If this function is monotone decreasing, one can be assured of stability over time, towards a fixed point

$$\frac{dL}{dt} = \left(\frac{\partial L}{\partial \mathbf{x}_i} \right) \dot{\mathbf{x}}_i + \left(\frac{\partial L}{\partial \mathbf{v}_i} \right) \dot{\mathbf{v}}_i$$

$$\frac{dL}{dt} = -\sigma \sum_i |\mathbf{v}_i|^2 \leqslant 0$$

We only discussed the base case. More details for obstacles but same technique.