Structure and Synthesis of Robot Motion: Tutorial 2 (Semester 2 - 2011/12)

Subramanian Ramamoorthy and Benjamin Rosman

Questions:

1. [Savage's Omelet Problem]
   The eminent statistician Savage poses a problem based on the following scenario. "Your friend has just broken five good eggs into a bowl when you come in and volunteer to finish making the omelet. A sixth egg, which for some reason must be either used for the omelet or wasted altogether, lies unbroken beside the bowl. You must decide what to do with this unbroken egg. Perhaps it is not too great an oversimplification to say that you must decide among three acts only, namely, to break it into the bowl containing the other five, to break it into a saucer for inspection, or to throw it away without inspection. Depending on the state of the egg, each of the three acts will have some consequence of concern to you".
   Formulate this as a decision problem involving states, actions and utilities - attributing costs and utilities as you see fit - and obtain the optimal course of action based on a utility maximization argument. Carefully work out and think about each step along the way, and assumptions being made. Discuss these with your tutor.

2. Figure 1 shows an Influence Diagram pertaining to the decision to test an infant born to a mother who is infected with the HIV (human immunodeficiency virus) with the PCR (polymerase chain reaction, a technique that is useful for HIV diagnosis in infants).
   Convert it into a decision tree form.

3. Consider the Rock-Paper-Scissors problem, where each player simultaneously utters one of three words *rock*, *paper* or *scissors*. If both players utter the same word, the game is a draw. Otherwise, one player wins 1 gold coin from the other player according to the following: Scissors defeat (cuts) paper, paper defeats (covers) stone, and stone defeats (breaks) scissors.
   Pose this as a matrix game.

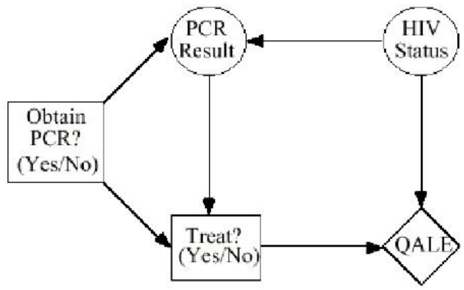

Figure 1: Influence Diagram for the HIV treatment decision.

Can you find a solution to the game via an iterated dominance argument? Pose the solution of this problem, i.e., best course of action from one player's point of view, as a linear program and obtain the optimal strategy.

4. Determine the set of all mixed saddle-point solutions of the following two-player matrix games, where Player 1 (minimiser) picks a row and Player 2 (maximiser) picks a column. The essential line of reasoning is the familiar minimax argument but note that you may need a 'mixed strategy', where primitive actions are combined according to a probability of choosing them.

   (a) $\begin{pmatrix} 1 & 3 & -1 & 2 \\ -3 & -2 & 2 & 1 \\ 0 & 2 & -2 & 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 2 & 1 & 0 & -1 \\ -1 & 3 & 1 & 4 \end{pmatrix}$

5. Consider the Lady in the Lake pursuit evasion game mentioned in lectures. Experiment with a restricted and discrete version of this game as follows.
   During each time step, both players get to make a fixed movement along a chosen direction (i.e., instead of moving with an instantaneous velocity of v, the player is forced to make a movement of vT for a fixed time time interval T). Let us thus assume that P can cover a distance along the circumference of 0.1 unit in one time step while E can only cover a distance $0.025 < z < 0.075$.
   Implement an optimization process (using a function like Matlab's fmin-

search) for a fixed time horizon of 25 time steps to synthesize motion sequences for P and E (i.e., a vector 25 elements long for each player representing the decisions taken for each time step). Try this for a number of different initializations of the path, including at least 2-3 random vectors as seed points.
How do your results compare against the theoretical optimum, i.e., the figure in lecture slides? Discuss similarities and differences.