Structure and Synthesis of Robot Motion: Coursework Assignment 3 (Semester 2 - 2011/12)

Subramanian Ramamoorthy and Benjamin Rosman

Instructions:

1. This homework assignment is to be done *individually*, without help from your classmates or others. Plaigarism will be dealt with strictly as per University policy.
2. Solve all problems and provide your **complete** solutions (with adequate reasoning behind each step) in a computer-printed or *legibly* handwritten form.
3. For computational questions, include an appendix with your code (e.g., Matlab commands) and all major numerical parameters involved.
4. The three questions carry equal weight. It is due at 4 pm on 22 March 2012.

Questions:

1. [Automated Driving]
   Consider the following problem faced by an autonomous driving robot. This robot is driving on the left most lane at a constant speed, v. To its right, in an adjoining lane, is a human driver with a lot on her mind. In front of the robot is another car that is driving at an erratic speed $v_f \sim N(v, 0.25v)$.

   Typically, human drivers use an indicator signal when they wish to change lanes but sometimes they do not (and just nudge in, which is apparent when they 'seem' to deviate from a straight path). Moreover, it is often observed that human drivers only do this nudging if they believe there is a two-car length space free in the left most lane. Also, just before they nudge in, they speed up slightly.

   The problem faced by our robot is that it only knows what is happening on the road through some noisy visual sensors. So, it receives a noisy estimate of the (x, y, θ) position of the nearby cars, from which they obtain a correspondingly noisy estimate of velocity. Similarly, people only maintain a noisy representation in their heads of exactly how much space is free on each lane.

As far as our robot is concerned, a collision is bad.

Your tasks are as follows:

(a) With all of these considerations in mind, state the decision problem in precise terms, using symbols for states, actions, utilities, etc. in particular, formulate the motion strategy of the automated car in terms of an influence diagram that includes all sources of uncertainty and choice. You are free to decide on the assignment of free variables (but justify such choices).

(b) Assign a specific strategy to the cars in front and beside the automated car (i.e., turn their decisions into probabilities based on observables). Using this, recast your decision problem for the robot as a decision tree.

(c) Work out, by hand, an instance of the decision for a specific time instant.

2. Consider the Rock-Paper-Scissors problem, where each player simultaneously utters one of three words *rock*, *paper* or *scissors*. If both players utter the same word, the game is a draw. Otherwise, one player wins 1 gold coin from the other player according to the following: Scissors defeat (cuts) paper, paper defeats (covers) stone, and stone defeats (breaks) scissors.
Write this as a matrix game and as an influence diagram.

Consider a specific opponent against whom you get to play many instances of this game. This person plays a mixed strategy that assigns probabilities, $(p_r = 0.25, p_p = 0.4, p_s = 0.35)$. Assigning this strategy to your opponent, within the influence diagram, generate a decision tree and an optimal strategy for playing this game.

However, in general, you do not know the opponent's strategy ahead of time.

One way to approach this is by adopting the *fictitious play* procedure:
Assume at each period that the opponent is using a stationary mixed strategy. Choose actions to maximize that period's expected payoff given the prediction of the distribution of opponents' actions, which you form according to the empirical frequency.

Simulate this play for 5000 rounds and plot the payoffs received.

Now, consider the following matrix game:

$$\begin{pmatrix} (0,1) & (0,0) \\ (0,0) & (0,1) \end{pmatrix} \tag{1}$$

Repeat the above against an opponent who plays the two actions with probability $(p, 1-p)$ and simulate fictitious play for 1000 steps, for $p = 0.5, 0.25, 1.0$.

3. Consider the following model for how a rumour (or other piece of information) spreads through a closed population (of size $N + 1$) of agents. At time t, the total population can be classified into three categories:

   - x persons who are ignorant of the rumour;
   - y persons who are actively spreading the rumour;
   - z persons who have heard the rumour but have stopped spreading it: if two persons who are spreading the rumour meet then they stop spreading it.

   The contact rate between any two categories is a constant μ. The following is the deterministic model of the problem:

$$\dot{x} = -\mu xy \tag{2}$$
$$\dot{y} = \mu[xy - y(y-1) - yz] \tag{3}$$

   Note that for a fixed population size, $z = (N + 1 - x - y)$.

   (a) Generate and sketch the trajectories in phase space for this system using Matlab. Use a value of $\mu = 0.75$. [Note: In order to do that, select a grid of initial values in the $x - y$ phase space and iterate (i.e., solve initial value problems) over small time steps - I suggest 0.25 seconds (time units) - using the above differential equations to visualize the state evolution. You may use a solver like ode45 in Matlab].

   (b) Starting with the initial conditions $y = 1$ and $x = N = 75$, plot $x(t)$ and $y(t)$ over a time interval of 3 seconds/time units.

   (c) Change the free parameter μ and discuss the corresponding change in behaviour.