SPNLP Level 10:
Assignment 1

Issued on Monday 14th January; due back **4.00pm** on Monday 11th February

Submission

This assignment should be submitted in hard copy format. It is acceptable to include handwritten formulas and diagrams, as long as these are in a clear, readable format. In order to preserve anonymity, you may identify your work using only your exam number, although this is not obligatory. Please ensure that every page is clearly numbered and that your exam number or matriculation number appears at the top of every page. ***Please submit your work using the submission box in the foyer of Appleton Tower Level 4.*** *The cut-off time for submission is 4.00pm, Monday 11th February.*

First-Order Logic

Exercise 1:
List all the subformulas (i.e., subexpressions of type **Term**(BOOL)) of the following formula. For each subformula ϕ, give the set $\textsc{free}(\phi)$ of free variables in ϕ. You should assume that a is a constant rather than a variable.

(1) $(((Q\,x) \lor \exists x.\forall y.((P\,z\,x) \land (Q\,a))) \lor \forall z.(R\,x\,z\,x))$

Exercise 2:
Using the `nltk.sem` module, construct a model that jointly satisfies the following sentences. If you can't construct a model that jointly satisfies all of them, describe why not. Translate the sentences into a form that can be parsed by `LogicParser` and use the `evaluate()` function to show the semantic value that each sentence receives in your model. Give a print-out of this evaluation.

(2) $\forall x.\exists y.(\mathit{neighbour}\,y\,x)$

(3) $\forall x.(\neg(\mathit{neighbour}\,x\,x))$

(4) $\forall x.\forall y.\forall z.(((\mathit{neighbour}\,y\,x) \land (\mathit{neighbour}\,z\,y)) \rightarrow (\mathit{neighbour}\,z\,x))$

Exercise 3:
This exercise is meant to remind you that the *names* for predicates are meaningless, in the sense that the predicates receive their interpretation solely from the valuation function of the model.

Devise a model for each of the following formulas, and then think about real-world concepts that could be 'translated' with the predicates P, Q_1 and Q_2. That is, think about natural language sentences that fill in the predicates and inuitively make the formula on the right side of the '$\rightarrow$' true when that on the left is true.

(5) a. $\forall x.\forall y.((P\, y\, x) \rightarrow (P\, x\, y))$
b. $\forall x.\forall y.((Q_1\, y\, x) \rightarrow (Q_2\, x\, y))$

Exercise 4:
Consider the following description:

> *There are exactly four blocks. Two of the blocks are cubical, and two are pyramid shaped. The cubical blocks are the same size, and both are red. The larger of the two pyramids is green, the smaller is yellow. Three of the blocks are sitting directly on the table, while the small pyramid is sitting on a cube.*

(a) Devise a suitable vocabulary of non-logical constants and represent the described situation as a first order model.

(b-i) Describe this situation in a first order language using the vocabulary from (a). That is, give a set of first order formulas that is true only in models like the one you gave for (a). (Note that this doesn't necessarily mean that you have to 'translate' every sentence; all you're asked to do is to describe the situation that is described in English above.)

To get you started, here is how you can say "there are exactly four blocks."

(6)
$$\begin{aligned}\exists x_0.\exists x_1.\exists x_2.\exists x_3.(&(\mathit{block}\, x_0) \wedge \ldots \wedge (\mathit{block}\, x_3) \wedge \\ &(x_0 \neq x_1) \wedge \ldots \wedge (x_2 \neq x_3) \wedge \\ &\forall y.((\mathit{block}\, y) \rightarrow ((y = x_0) \vee \ldots \vee (y = x_3))))\end{aligned}$$

(b-ii) Can you think of models for this set of formulae which differ from the one you gave for (a) and still satisfy the set?

Semantic Composition

Exercise 5:
The following command will load a simple-minded feature-based grammar from the NLTK distribution:

```
grammar = nltk.data.load('grammars/sem2.fcfg')
```

This grammar contains a grammar rule for transitive verb phrases and a lexical entry for the verb *chases*, as follows:

```
VP[num=?n,sem=<app(?v,?obj)>] -> TV[num=?n,sem=?v] NP[sem=?obj]

TV[num=sg,sem=<\X y. (X \x. (chase x y))>,tns=pres] -> 'chases'
```

(Note that `X` represents a variable of the same type as quantifier phrases; i.e., ((IND $\rightarrow$ BOOL) $\rightarrow$ BOOL).)

Extend the grammar with a rule for ditransitive verbs and a lexical entry for *show* which will capture sentences such as:

(7) Suzie shows John a dog.

In addition, build a model which contains a valuation for *show*, and demonstrate that it satisfies semantic representations of sentences containing this verb.

Exercise 6:
Translate the following sentence into FOL:

(8) Every person needs a doctor.

Build a model that only satisfies one reading. Devise another model that satisfies both readings. What does this tell you about the semantic relationship between the two readings? (You can choose whether or not to implement the model in NLTK.)

Tableaux Proof Methods

Exercise 7:
The connective *nor* is defined by the following truth tables:

p	q	p *nor* q
TRUE	TRUE	FALSE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	TRUE

(a) Give the tableaux expansion rules for this connective.

(b) Show that $(\neg(p \vee q)) \leftrightarrow (p \text{ nor } q)$ is provable.