Reinforcement Learning (INF11010)

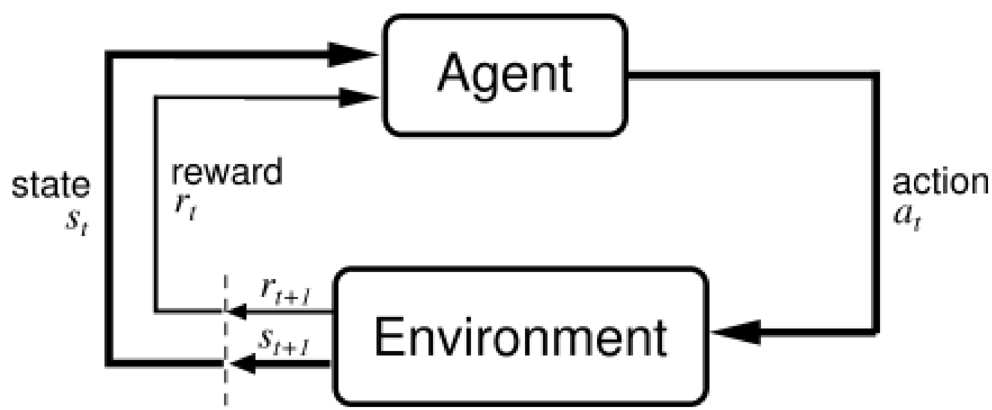
Lecture 6: Dynamic Programming for Reinforcement Learning (extended)

Pavlos Andreadis, February 2nd 2018

Markov Decision Processes

- A finite Markov Decision Process (MDP) is a tuple (S, A, P, R, γ) where:

- S is a finite set of states
- A is a finite set of actions
- P is a state transition probability function
- R is a reward function
- γ is a discount factor



Today's and Friday's Content

- Dynamic Programming (DP) solutions to the RL problem
- Policy Evaluation + Policy Improvement →
  Policy Iteration || Value Iteration
- Backup diagrams and the Bellman Equation
- Generalised Policy Iteration
- Asynchronous Dynamic Programming
- Dynamic Programming methods in relation to other approaches

Dynamic Programming

- Algorithms for optimal policies given a *perfect model* of the environment as a Markov decision process (MDP)
- … but of theoretical importance.

- Applicable for exact solutions with discrete state & action model:

$$P^a_{s,s'} = Pr\{s_{t+1} = s' | s_t = s, a_t = a\}$$

$$R^a_{s,s'} = E\{r_{t+1} | a_t = a, s_t = s, s_{t+1} = s'\}$$

- … and provide approximate solutions for continuous problems.

Bellman Optimality Equations

$$V^*(s) = max_a E\{r_{t+1} + \gamma\, V^*(s_{t+1}) | s_t = s, a_t = a\}$$
$$= max_a \sum_{s'} P^a_{ss'} \left[R^a_{ss'} + \gamma\, V^*(s') \right]$$

$$Q^*(s,a) = E\left\{ r_{t+1} + \gamma\ max_{a'} Q^*(s_{t+1}, a') | s_t = s, a_t = a \right\}$$
$$= \sum_{s'} P^a_{ss'} \left[R^a_{ss'} + \gamma\ max_{a'}\ Q^*(s', a') \right]$$

for all $s \in S, a \in A(s),$ and $s' \in S.$

Policy Evaluation

- There exists a unique solution as long as $\gamma < 1$ or termination is guaranteed:

$$\begin{aligned} V^\pi(s) &= E_\pi\{r_{t+1} + \gamma\, r_{t+2} + \gamma^2\, r_{t+3} + ... | s_t = s\} \\ &= E_\pi\{r_{t+1} + \gamma V^\pi(s_{t+1}) | s_t = s\} \\ &= \sum_a \pi(s,a) \sum_{s'} P^a_{ss'} \left[R^a_{ss'} + \gamma V^\pi(s')\right] \end{aligned}$$

- … which is a system of |S| linear equations with |S| unknowns

Iterative Policy Evaluation

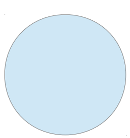
- An iterative solution, starting from an arbitrary V_0 (but with terminal states having a value of 0) and computing…

$$
\begin{aligned}
V_{k+1}(s) &= E_\pi\{r_{t+1} + \gamma V_k(s_{t+1}) | s_t = s\} \\
&= \sum_a \pi(s,a) \sum_{s'} P^a_{ss'} \left[R^a_{ss'} + \gamma V_k(s') \right]
\end{aligned}
$$

- … which converges to V^π as $k \to \infty$
- At every iteration, every state is *backed up*
- For DP, this is a *full backup*, since we don't sample next states

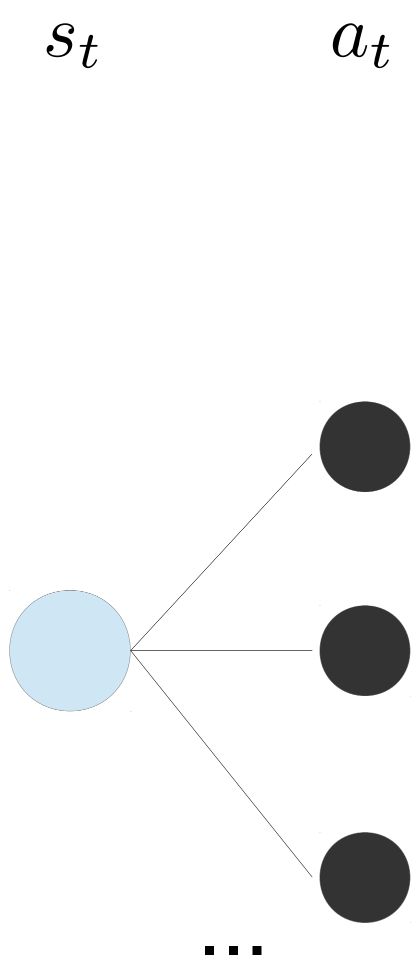
Backup Diagrams

s_t



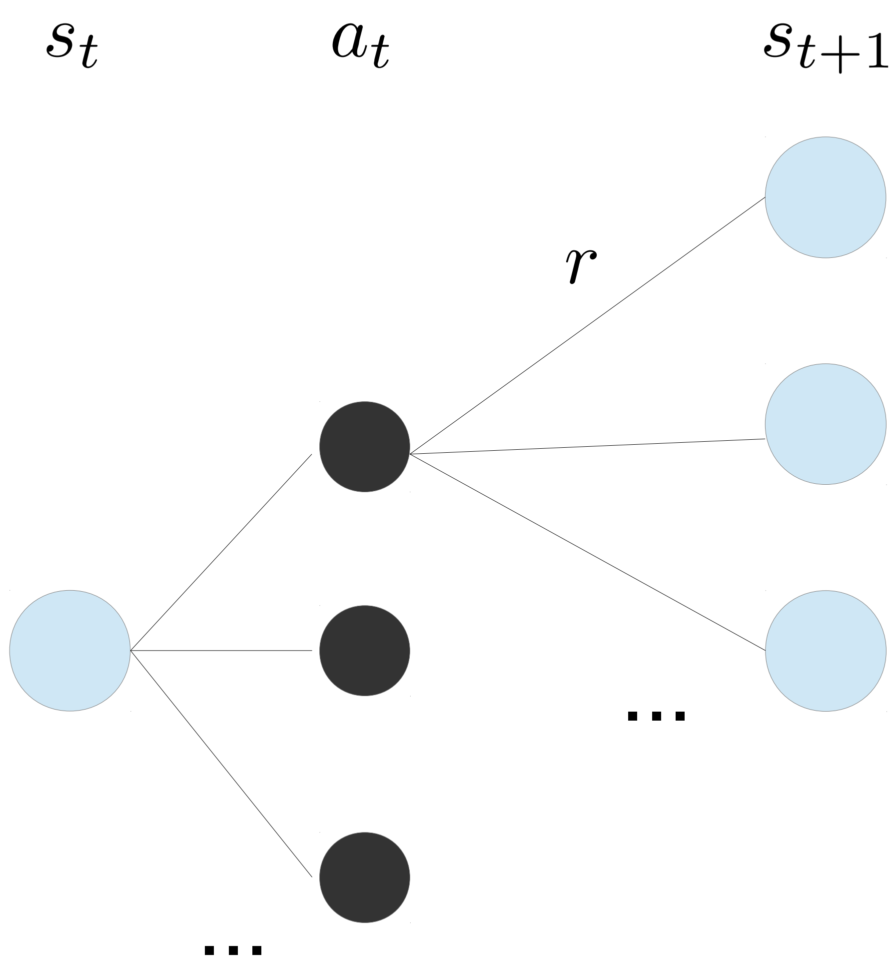
- State value function V

Backup Diagrams



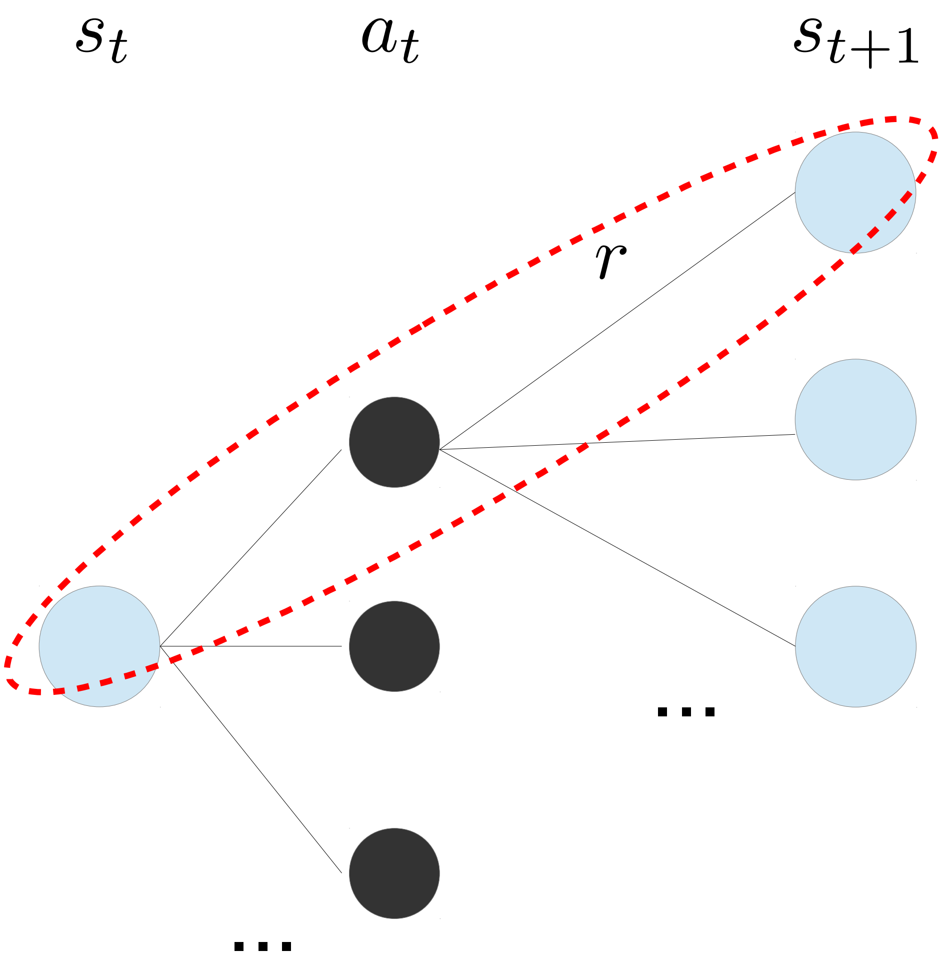
- State value function V

Backup Diagrams



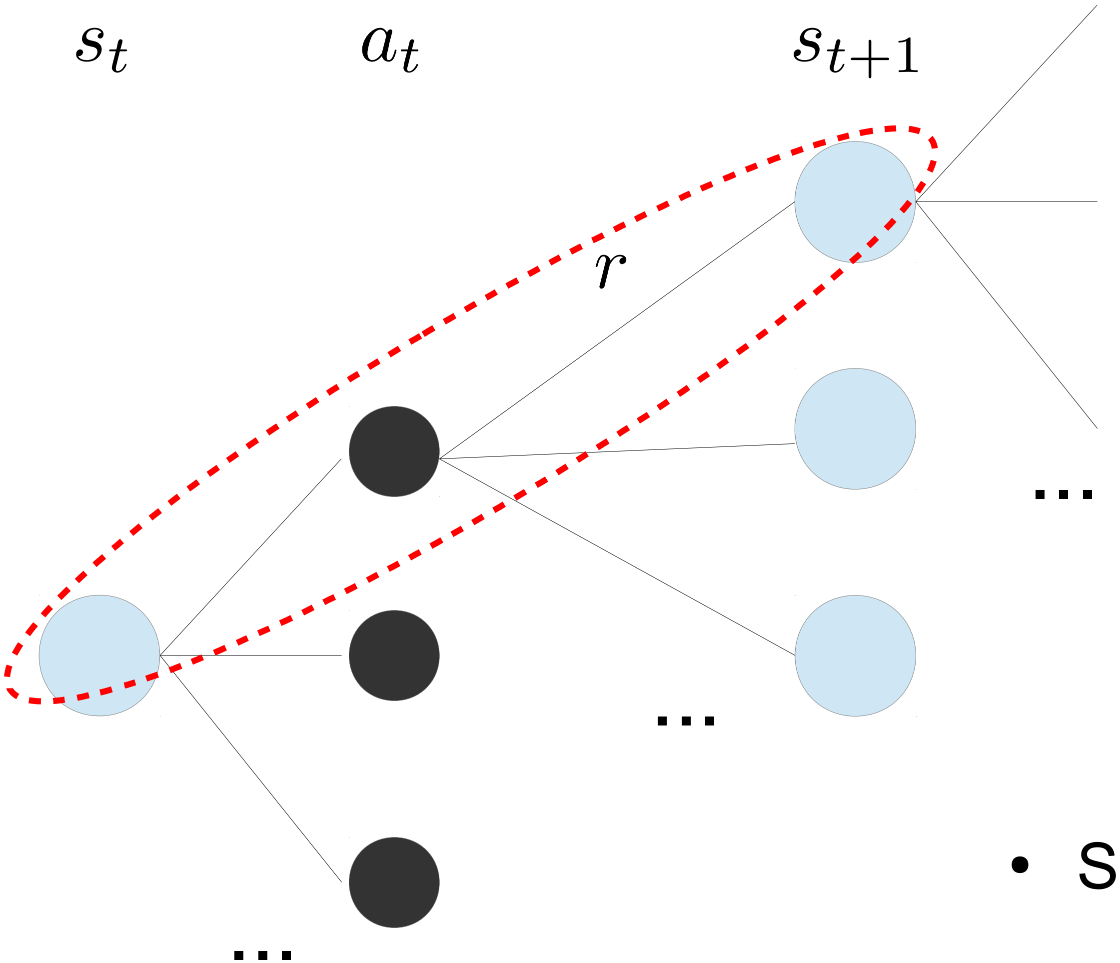
- State value function V

Backup Diagrams



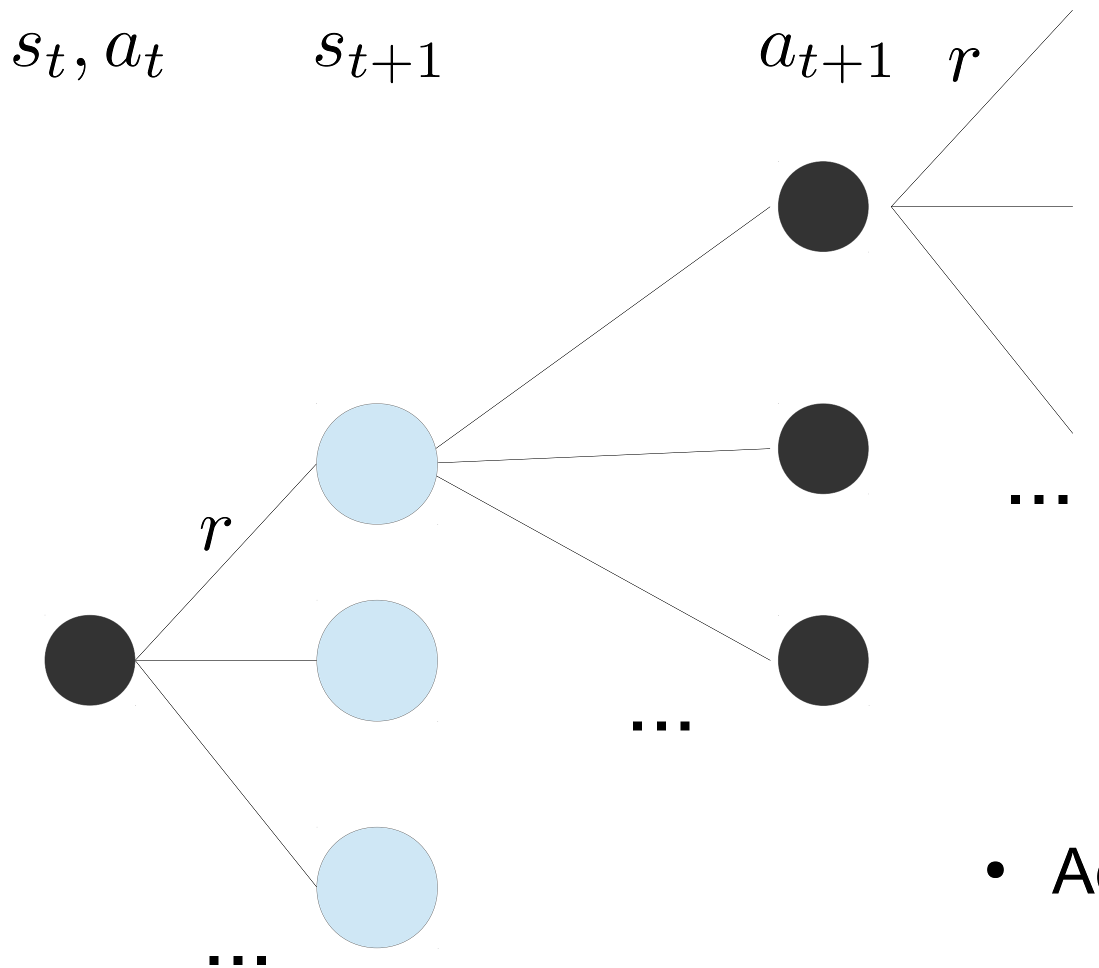
- State value function V

Backup Diagrams



- State value function V

Backup Diagrams



- Action value function Q

Policy Improvement

- Consider a given policy π
  - … can we improve it by changing the action taken at a specific state s ?
  - … yes if $Q^\pi(s,a) > V^\pi(s)$

- *[Policy Improvement Theorem]* Generally, for deterministic policies π , π', if

$$Q^\pi(s,\pi'(s)) \geq V^\pi(s),\ \forall s \in S$$

then

$$V^{\pi'}(s) \geq V^\pi(s),\ \forall s \in S$$

greedy Policy Improvement

- A policy improvement step would then be:

$$\begin{aligned}\pi'(s) &= argmax_a Q^{\pi}(s,a) \\ &= argmax_a \sum_{s'} P^a_{ss'}\left[R^a_{ss'} + \gamma\, V^{\pi}(s')\right]\end{aligned}$$

- Of course, this does not evaluate the value function for the new policy π', but if we put Policy Improvement and Policy Evaluation together, we get...

Policy Iteration

1. initialise V and π_0 (arbitrarily)

2. perform Policy Evaluation

3. perform Policy Improvement

4. if the policy has changed go to 2.

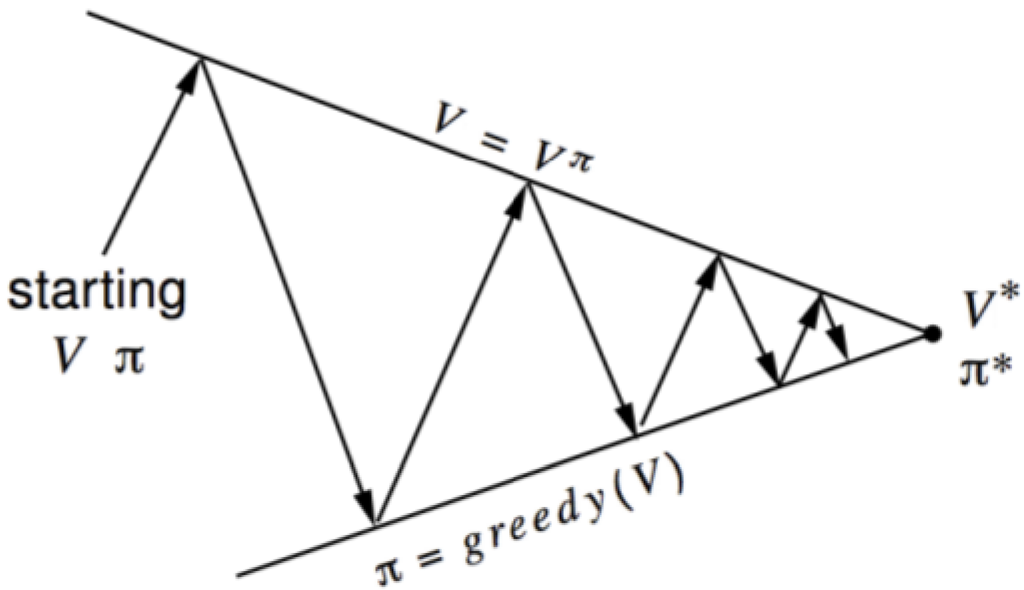
Value Iteration

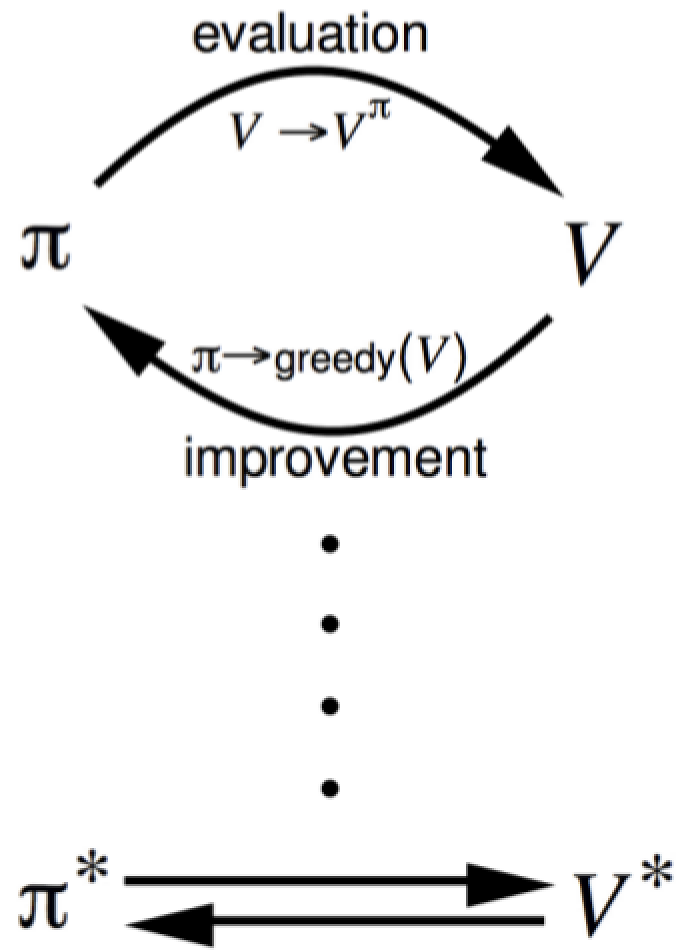
- … is like Policy Iteration but with only a single backup of each state in the Policy Evaluation step.
- This still converges to an optimal policy.
- Policy Evaluation and Policy Improvement can be joined into a single update:

$$\begin{aligned} V_{k+1}(s) &= max_a E\{r_{t+1} + \gamma\, V_k(s_{t+1}) | s_t = s, a_t = a\} \\ &= max_a \sum_{s'} P^a_{ss'} \left[R^a_{ss'} + \gamma V_k(s') \right] \end{aligned}$$

- Need only compute the policy in the end.

Policy Iteration (concept)

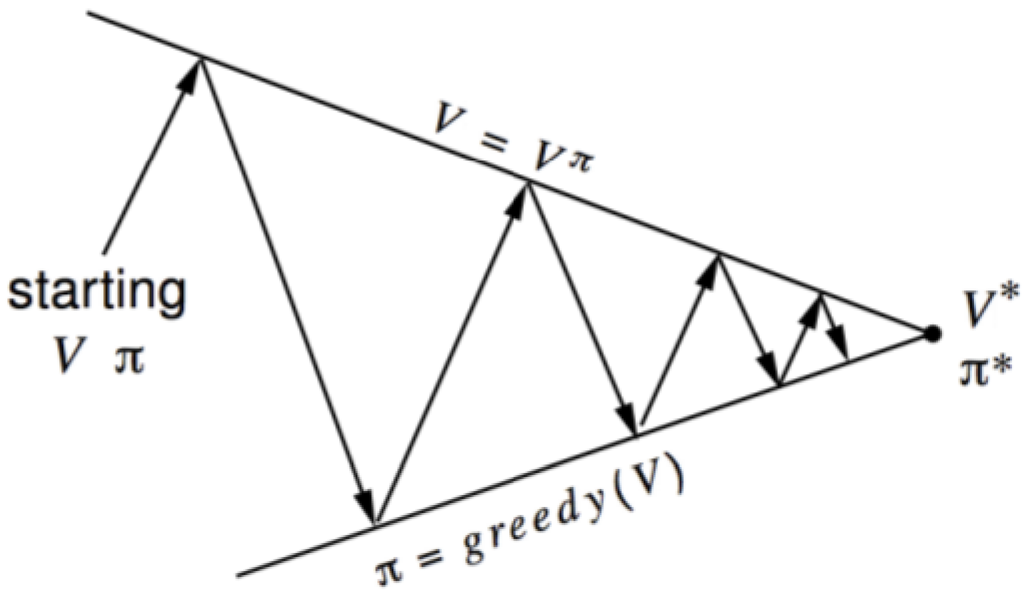


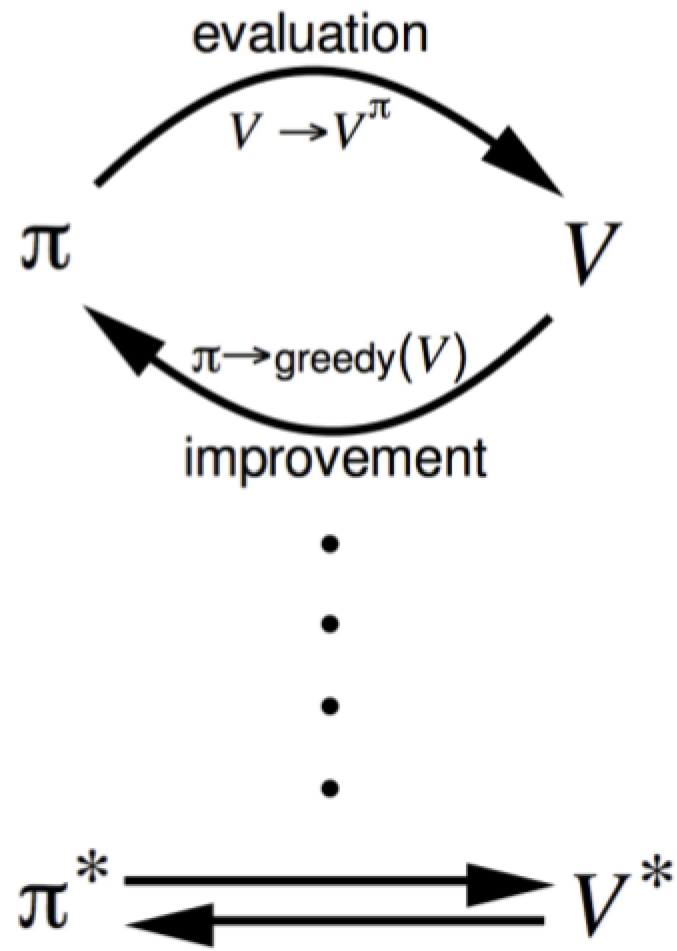


Policy evaluation Estimate v_π
Iterative policy evaluation

Policy improvement Generate $\pi' \geq \pi$
Greedy policy improvement

Generalised Policy Iteration





Policy evaluation Estimate v_π

Policy improvement Generate $\pi' \geq \pi$

Asynchronous DP

- So far, all solutions have considered full sweeps of the state space.
- An Asynchronous DP procedure performs evaluation & improvement computations ...
  - without going through all the states or in any specific ordering
  - with any state or state-action values currently available
- To converge correctly, it needs to visit all states in expectation.
- These are not necessarily faster, but depending on the problem might help us improve convergence by e.g. avoiding states that do not appear in optimal trajectories.
- Examples:
  - Value iteration with only one state updated per iteration.
  - Real-time dynamic programming

DP Efficiency

- Value Iteration for V (1 iteration) has complexity $O(|S|^2 \; |A|)$.
- Value Iteration for Q (1 iteration) has complexity $O(|S|^2 \; |A|^2)$.

- Policy Iteration…
  - takes time polynomial in the problems size (state and action space)
  - converges much slower the closer γ is to 1

- 1 iteration of *Policy Iteration* is slower than 1 iteration of *Value Iteration*, but Policy Iteration will generally require fewer iterations till convergence.

DP in Comparison to Other Methods

- Dynamic Programming Methods:
  - *require a model*
  - *bootstrap*
- Monte Carlo Methods:
  - *do not* require a model
  - *do not* bootstrap
- Temporal-Difference Learning Methods:
  - *do not* require a model
  - *bootstrap*

Reading +

- Chapter 4 of Sutton and Barto (1st Edition)
  http://incompleteideas.net/book/ebook/the-book.html

- Please join Piazza for announcements and support:
  https://piazza.com/ed.ac.uk/spring2018/infr11010

Optional:

- Littman, M. L. and Dean, T. L. and Kaelbling, L. P. (1995)
  On the complexity of solving Markov decision problems.

- Pashenkova, E. and Rish, I. and Dechter, R. (1996)
  Value iteration and policy iteration algorithms for Markov decision problem.