Randomness and Computation

or, “Randomized Algorithms”

Heng Guo

(Based on slides by M. Cryan)

Coupon Collector Problem

> *“Coupon collecting” is the activity of buying packs, each of which will have a uniform at random coupon inside. There are be n different types of “coupon” and the goal is to collect one copy of each ... then stop buying.*

Last time we have showed that the expected number $\mathrm{E}[X]$ of purchases to collect all cards is $nH(n) \sim n\ln(n)$.

Today we examine how likely a example “run” of the purchasing process is to come close to that expectation.

Concentration inequalities will be vital:

- Markov Inequality;
- Chebyshev Inequality;
- Chernoff Bound / Hoeffding inequality.

Markov Inequality

The simplest one.

Theorem (3.1, Markov Inequality)

Let X be any random variable that takes only non-negative values. Then for any $a > 0$,

$$\Pr[X \geq a] \leq \frac{E[X]}{a}.$$

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Then $X \geq a \cdot I(X)$, and hence $I(X) \leq \frac{X}{a}$.

Taking expectation of both sides, and using $\mathrm{E}[I] = \Pr[X \geq a]$, we have

$$\Pr[X \geq a] \;=\; \mathrm{E}[I] \;\leq\; \frac{1}{a}\mathrm{E}[X]. \qquad \square$$

Bounding Coupon Collector purchases - Markov

Recall that X is the number of purchases of the coupon collector problem and $\mathrm{E}[X] = n \ln n + \Theta(n)$.

Say we want a bound T so that the probability of $X \geq T$ is at most $\frac{1}{n}$.

By Markov ineq., $\Pr[X \geq T] \;\leq\; \frac{\mathrm{E}[X]}{T}$. Thus, we need T to be at least $n^2 \ln n$.

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This is far from tight!

The power of Markov ineq. is that it does not require any other knowledge of the random variable. However for specific problems, we can often do better.

For example, we can bound the variance.

Variance, Moments of a Random Variable

Definition (3.1)

The *kth moment* of a random variable X is defined to be $\mathrm{E}[X^k]$.

Definition (3.2)

The *variance* of a random variable is defined to be

$$\mathrm{Var}[X] \;:=\; \mathrm{E}[(X-\mathrm{E}[X])^2] \;=\; \mathrm{E}[X^2]-\mathrm{E}[X]^2.$$

Covariance of two Random Variables

Definition (3.3)

The *covariance* of two random variables X and Y is defined as

$$\mathrm{Cov}[X,Y] \;=\; \mathrm{E}\left[(X-\mathrm{E}[X])(Y-\mathrm{E}[Y])\right].$$

Theorem (3.2)

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$$Var[X+Y] \;=\; Var[X]+Var[Y]+2Cov[X,Y].$$

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$$\begin{aligned}\mathrm{Var}[X+Y] &= \mathrm{E}[(X+Y)^2]-\mathrm{E}[X+Y]^2\\ &= \mathrm{E}[X^2]+\mathrm{E}[Y^2]+2\mathrm{E}[XY]-\mathrm{E}[X]^2-\mathrm{E}[Y]^2-2\mathrm{E}[X]\mathrm{E}[Y]\\ &= \mathrm{Var}[X]+\mathrm{Var}[Y]+2(\mathrm{E}[XY]-\mathrm{E}[X]\mathrm{E}[Y]).\end{aligned}$$

$\square$

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Proof.

$$\begin{aligned} \mathrm{Var}[X+Y] &= \mathrm{Var}[X] + \mathrm{Var}[Y] + 2(\mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y]). \\ \mathrm{Cov}[X,Y] &= \mathrm{E}[XY - \mathrm{E}[X]Y - \mathrm{E}[Y]X + \mathrm{E}[X]\mathrm{E}[Y]] \\ &= \mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y] - \mathrm{E}[X]\mathrm{E}[Y] + \mathrm{E}[X]\mathrm{E}[Y] \\ &= \mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y] \end{aligned}$$

□

(pairwise) Independent Random Variables

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Corollary (3.4)

If X, Y are a pair of independent random variables, then

$$Cov[X, Y] \;=\; 0$$

and

$$Var[X + Y] \;=\; Var[X] + Var[Y].$$

Proof is straightforward application of Thm 3.3.

Chebyshev Inequality

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For every $a > 0$,

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$$\Pr[(X - \mathrm{E}[X])^2 \geq a^2] \;\leq\; \frac{\mathrm{E}[(X - \mathrm{E}[X])^2]}{a^2},$$

and by definition of $\mathrm{Var}(\cdot)$, this gives

$$\Pr[|X - \mathrm{E}[X]| \geq a] \;=\; \Pr[(X - \mathrm{E}[X])^2 \geq a^2] \;\leq\; \frac{\mathrm{Var}[X]}{a^2}. \qquad \square$$

Bounding Coupon Collector purchases - Markov

Recall that X is the number of purchases of the coupon collector problem and $\mathrm{E}[X] = n \ln n + \Theta(n)$.

Using Markov ineq., we can get an upper bound of a “typical” number of the order $n^2 \ln n$, which is not particularly interesting.

We can do better with Chebyshev ineq. . . .

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- Corollary 3.4: for independent Y, Z, $\mathrm{Var}[Y + Z] = \mathrm{Var}[Y] + \mathrm{Var}[Z]$.
  Are these X_i's independent?

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- X_i is independent of the value of X_{i-1} or any of the earlier values. X_i only depends on the values n and i (and not on what cards we have collected or how long it takes to collect them).
- Hence the random variables $X_1, \ldots, X_n$ are all mutually independent, and
  $$\mathrm{Var}[X] \;=\; \mathrm{Var}[X_1] + \mathrm{Var}[X_2] + \ldots + \mathrm{Var}[X_n].$$

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$$
\begin{aligned}
\mathrm{E}[X^2] &= \Pr[Y=1]\mathrm{E}[X^2 \mid Y=1] + \Pr[Y=0]\mathrm{E}[X^2 \mid Y=0] \\
&= p + (1-p)\sum_{x\geq 1} x^2 \Pr[X = x \mid Y=0] \\
&= p + (1-p)\sum_{x\geq 2} x^2 \Pr[X = x-1] \\
&= p + (1-p)\sum_{x\geq 1} (x+1)^2 \Pr[X = x] = p + (1-p)\mathrm{E}[(X+1)^2]
\end{aligned}
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Proof (cont'd).

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Proof (cont'd).

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\begin{aligned}
\mathrm{E}[X^2] &= p + (1-p)\mathrm{E}[(X+1)^2] \\
&= p + (1-p)\mathrm{E}[X^2] + 2(1-p)\mathrm{E}[X] + 1 - p \\
&= 1 + \frac{2(1-p)}{p} + (1-p)\mathrm{E}[X^2].
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We can solve that $\mathrm{E}[X^2] = \frac{2-p}{p^2}$. Thus,

$$\mathrm{Var}[X] = \mathrm{E}[X^2] - \mathrm{E}[X]^2 = \frac{1-p}{p^2}. \qquad \square$$

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$$\Pr[|X - \mathrm{E}[X]| \geq a] \;\leq\; \frac{\mathrm{Var}[X]}{a^2} \;=\; \frac{\sum_{j=1}^{n} \mathrm{Var}[X_j]}{a^2}.$$

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Each individual X_j is geometric with parameter $\frac{n-(j-1)}{n}$, so each X_j has

$$\mathrm{Var}[X_j] \;=\; \frac{j-1}{n}\left(\frac{n}{(n+1-j)}\right)^2 \;\leq\; \left(\frac{n}{n+1-j}\right)^2.$$

RC (2019/20) – Lecture 5 – slide 13

Bounding Coupon Collector purchases - Chebyshev

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Hence, using the Euler's series for $\frac{\pi^2}{6}$,

$$\mathrm{Var}[X] \;\leq\; n^2 \sum_{j=1}^{n} \left(\frac{1}{n+1-j}\right)^2 = n^2 \sum_{j=n}^{1} \left(\frac{1}{j}\right)^2 \;\leq\; \frac{\pi^2 n^2}{6}.$$

RC (2019/20) – Lecture 5 – slide 13

Bounding Coupon Collector purchases - Chebyshev

We know $\mathrm{Var}[X] \le \frac{\pi^2 n^2}{6}$ for our coupon collector process.

Bounding Coupon Collector purchases - Chebyshev

We know $\mathrm{Var}[X] \le \frac{\pi^2 n^2}{6}$ for our coupon collector process.

Suppose we are willing to make $2\mathrm{E}[X]$ (about $2n\ln(n)$) purchases.

The probability we fail to get all cards is

$$\begin{aligned} \Pr[X > 2\mathrm{E}[X]] &= \Pr[X - \mathrm{E}[X] > \mathrm{E}[X]] \\ &= \Pr[|X - \mathrm{E}[X]| > \mathrm{E}[X]]. \qquad \text{(as } X \ge 0\text{)} \end{aligned}$$

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We can upper bound the probability of the bad event $|X - \mathrm{E}[X]| > \mathrm{E}[X]$ using Chebyshev Inequality with $a = \mathrm{E}[X]$:

$$\begin{aligned} \Pr[|X - \mathrm{E}[X]| \ge \mathrm{E}[X]] \le \frac{\mathrm{Var}[X]}{\mathrm{E}[X]^2} &\le \frac{\pi^2 n^2}{6n^2 H(n)^2} \\ &= \frac{\pi^2}{6H(n)^2} \le \frac{2}{\ln(n)^2}. \end{aligned}$$

This improves over $\frac{1}{2}$, which is what Markov gives us.

Bounding Coupon Collector purchases - Union bound

Theorem (1.2, Union bound)

Let $E_1, E_2, \ldots$ be a finite or countably infinite sequence of events,

$$\Pr\left[\bigcup_{i \ge 1} E_i\right] \le \sum_{i \ge 1} \Pr[E_i].$$

Similar to Markov ineq., there is almost no requirement to the union bound!

Bounding Coupon Collector purchases - Union bound

Let E_i be the "bad" event where card i is still missing at time T.

$$\Pr[E_i] \le \left(1 - \frac{1}{n}\right)^T.$$

Thus, by a union bound,

$$\Pr[X \ge T] = \Pr\left[\cup_{i\ge 1}^n E_i\right] \le n\left(1 - \frac{1}{n}\right)^T.$$

Bounding Coupon Collector purchases - Union bound

Once again we use $(1 - 1/n)^n \le 1/e$. If $T = (1+\varepsilon)n\ln n$,

$$n\left(1 - \frac{1}{n}\right)^T \le n\left(\left(1 - \frac{1}{n}\right)^n\right)^{(1+\varepsilon)\ln n}$$
$$\le n(e^{-1})^{(1+\varepsilon)\ln n} = n^{-\varepsilon}.$$

Thus, for example if $\varepsilon = 1$,

$$\Pr[X \ge 2n\ln n] \le n^{-1}.$$

As $\mathrm{E}[X] \ge n\ln n$,

$$\Pr[X \ge 2\mathrm{E}[X]] \le \Pr[X \ge 2n\ln n] \le n^{-1}.$$

Coupon collector bounds

$$\Pr[X \ge 2\mathrm{E}[X]] \le \frac{1}{2} \quad \text{(Markov)}$$
$$\Pr[X \ge 2\mathrm{E}[X]] \le \frac{2}{\ln(n)^2} \quad \text{(Chebyshev)}$$
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The stronger the bounds are, the more information we use.

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Chebyshev also gives (weak) lower bound. Using Chernoff bound for negatively correlated rv, one can show

$$\Pr[X \le (1-\varepsilon)(n-1)\ln n] \le e^{-n^{\varepsilon}}.$$

However this is beyond the scope of our course. Check out Chapter 9 and 10 of `https://arxiv.org/abs/1801.06733` if you are interested.

Wrapping up today

Next week we will continue the theme of "bounding deviation from the mean" by introducing some stronger concentration inequalities called Chernoff bounds/Hoeffding ineq.

First, on Friday (to give a break) we will look at a simple random algorithm to approximately calculate **Max** Cut, and show how to *derandomize* it.

- Coursework 1 will be available on Thursday.
- Tutorials are starting next week. The first tutorial sheet will also be available soon.