Probabilistic Modelling and Reasoning, Tutorial Question Sheet 7 (for Week 10)

School of Informatics, University of Edinburgh

Instructor: Amos Storkey
Prob. Sheet. ID: 27114.1

1. Use the Kalman filtering equations given in Barber's book (see Algorithm 24.1) to derive the example of the simple Kalman filter given in the lecture slides, i.e. that for

$$z_{t+1} = z_t + w_t \qquad w_t \sim N(0,1)$$

$$x_t = z_t + v_t \qquad v_t \sim N(0,1)$$

$$p(z_1) = N(0, \sigma_0^2)$$

in the limit $\sigma_0^2 \to \infty$ we find that

$$\mu_3 = \frac{5x_3 + 2x_2 + x_1}{8}.$$

[Hint: show that $V_1 = 1$, $V_2 = 2/3$, $V_3 = 5/8$.]

2. Consider the noisy oscillator given by

$$\mathbf{x} = \alpha R \mathbf{x} + \boldsymbol{\epsilon}$$

where R is a unitary rotation matrix through a small angle, and $\boldsymbol{\epsilon}$ is a very small isotropic Gaussian noise term. Initialise at the unit vector. What happens to this system over time if $\alpha = 1$? What if $\alpha < 1$? Draw an example picture.

3. What would the effect of modelling this system with the wrong rotation matrix R be. How good a model would a (deterministic) circular motion be for this over the short term and over the long term?