Probabilistic Modelling and Reasoning, Tutorial Question Sheet 6 (for Week 9)

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Prob. Sheet. ID:12642.1

1. On Thursday the weather forecast for Saturday indicates a 60% chance of rain, and you are organizing an outdoor concert. The losses are as follows

$$\mathrm{L}(\text{go ahead}, \text{fair}) = -1 \qquad \mathrm{L}(\text{go ahead}, \text{rain}) = 2$$

$$\mathrm{L}(\text{cancel}, \text{fair}) = 3 \qquad \mathrm{L}(\text{cancel}, \text{rain}) = 0$$

   Calculate the minimum risk strategy. Should you cancel the concert?

2. A Hidden Markov Model problem. Consider a HMM with 3 states ($M = 3$) and 2 output symbols, with a left-to-right state transition matrix

$$A = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.0 & 0.6 & 0.4 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$$

   an output probabilities matrix

$$B = \begin{pmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \\ 0.8 & 0.2 \end{pmatrix}$$

   and an initial state probabilities vector $\pi = (0.9\ 0.1\ 0.0)$. Given that the observed symbol sequence $\mathbf{X}$ is 122, compute

   (i) $P(\mathbf{X})$

   (ii) $p(\mathbf{z}_2|\mathbf{X})$. [As there are 3 observations the HMM will have three time slices—you are asked to compute the posterior distribution of the state variable in the second time slice, numbering the times 1, 2, 3.] You can check this calculation by setting up the HMM in JavaBayes.

3. Suppose the matrix A in the HMM (qu 1.) had its rows all equal to the initial probabilities vector π. In this case the HMM reduces to a simpler model—what is it?

4. Show that if a transition probability a_{ij} in a HMM is set to zero initially, then it will remain at zero throughout training.

5. **(Extra question)**. Read about the Viterbi algorithm for finding the best state sequence given a sequence of observations, and apply it to the model in question 1.