Probabilistic Modelling and Reasoning, Tutorial Questions for Week 3

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1. [From Tipping, §2.1.3]. Box 1 contains 8 apples and 4 oranges. Box 2 contains 10 apples and 2 oranges. Boxes are chosen with equal probability. What is the probability of choosing an apple? (3/4). If an apple is chosen, what is the probability that it came from box 1? (4/9)

2. [R & N, Ex 14.5] Prove from first principles the conditionalized version of the general product rule

$$P(X, Y|Z) = P(X|Z)P(Y|X, Z).$$

Also prove the conditionalized version of Bayes' rule

$$P(X|Y, Z) = \frac{P(Y|X, Z)P(X|Z)}{P(Y|Z)}.$$

3. [Prosecutor's fallacy] This question concerns "DNA fingerprinting" evidence. The probability that there is a DNA match given that a person is innocent is estimated as $1/100,000$. Assume that the probability that there is a match given that a person is guilty is 1. Suppose that the defendant in a trial lives in a city where there are $10,000$ people who could have committed the crime, and that there is a DNA match to the defendant. Calculate $P(guilty|\ DNA\ match)$. How does this vary as the size of the population changes?

4. **(Bonus question) – the Three Prisoners problem**
(From Pearl, 1988) Three prisoners A, B and C are being tried for murder, and their verdicts will be read and their sentences executed tomorrow. They know only that one of them will be declared guilty and will be hanged while the other two will go free; the identity of the condemned prisoner is revealed to a reliable prison guard, but not to the prisoners.

In the middle of the night Prisoner A makes the following request. " Please give this letter to one of my friends – to one who is to be released. You and I know that at least one of them will be released.". The guard carries out this request. Later prisoner A calls the guard and asks him to whom he gave the letter. The guard tells him that he gave the letter to prisoner B. What is the probability that prisoner A will be released?

5. **(Bonus question) The Monte Hall problem**.
I have three boxes. In one I put a prize, and two are empty. I then mix up the boxes. You want to pick the box with the prize in it. You choose one box. I then open another one of the boxes and show that it is empty. I then give you the chance to change your choice of boxes—should you do so? How is this puzzle related to the Three Prisoners problem?