Answers to PMR tutorial questions (Week 4)

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1. Conditioned on s, marginalise out (b, g, t), and query f.

$$p(f = \text{empty}|s = \text{no}) = \frac{p(f = \text{empty}, s = \text{no})}{p(s = \text{no})} = \frac{p(f = \text{empty}, s = \text{no})}{\sum_f p(f, s = \text{no})} \tag{1}$$

Below we use the decomposition of the joint probability according to the DAG: $p(b, f, g, t, s) = p(b)p(f)p(t|b)p(g|b, f)p(s|t, f)$.

$$\begin{aligned}
p(f, s{=}\,\text{no}) &= \sum_{g,t,b} p(b, f, g, t, s{=}\,\text{no}) && \text{(marginalisation)}\\
&= \sum_{g,t,b} p(b)p(f)p(t|b)p(s = \text{no}|t, f)p(g|b, f) && \text{(factorisation)}\\
&= p(f)\sum_t p(s = \text{no}|t, f)\sum_b p(b)p(t|b)\sum_g p(g|b, f) && (2)\\
&= p(f)\sum_t p(s = \text{no}|t, f)\sum_b p(b)p(t|b) && (\textstyle\sum_g p(g|b, f) = 1)\\
&= p(f)\sum_t p(s = \text{no}|t, f)p(t) && (\text{as } \textstyle\sum_b p(b)p(t|b) = p(t))
\end{aligned} \tag{3}$$

Using the CPTs in the figure we have

$p(t = \text{no}) = p(b = \text{bad})p(t = \text{no}|b = \text{bad}) + p(b = \text{good})p(t = \text{no}|b = \text{good}) = 0.98 \times 0.02 + 0.03 \times (1 - 0.02) = 0.049$, $p(t = \text{yes}) = 1 - 0.049 = 0.951$.

Setting $f = \text{empty}$ we have

$$\begin{aligned}
p(f = \text{empty}, s = \text{no}) &= p(f = \text{empty})\sum_t p(s = \text{no}|t, f = \text{empty})p(t)\\
&= 0.05[1.0 \times 0.049 + 0.92 \times 0.951] = 0.0462
\end{aligned}$$

and for $f = \text{not empty}$

$$\begin{aligned}
p(f = \text{not empty}, s = \text{no}) &= p(f = \text{not empty})\sum_t p(s = \text{no}|t, f = \text{not empty})p(t)\\
&= 0.95[1.0 \times 0.049 + 0.01 \times 0.951] = 0.0556.
\end{aligned}$$

Thus

$$p(f = \text{empty}|s = \text{no}) = \frac{p(f = \text{empty}, s = \text{no})}{p(s = \text{no})} = \frac{0.0462}{0.0462 + 0.0556} = 0.4538.$$

2. "Given a set of conditioning nodes C, if *every* path from any node in set A to any node in set B is blocked, then A is said to be d-seperated from B by C. This implies $I(A, B|C)$."

"A path is said to be *blocked* if it includes a node such that either

1. the arrows on the path do not meet head-to-head at the node, and the node is in the conditioning set, or
2. the arrows do meet head-to-head, and neither the node, nor any of its descendents, is in the conditioning set."

1. $T - E - L - S$ is unblocked at E (head-to-head) because D is a descendent of E; unblocked at L (head-to-tail) because L is not in the evidence set. There exists a path that is unblocked, hence $\neg I(T, S|D)$.

2. (a) $L - E - D - B$ is blocked at D (head-to-head) because $D \notin \{S\}$ and S is not a descendent of D.
   (b) $L - S - B$ is blocked at S (tail-to-tail) because $S \in \{S\}$.
   All paths are blocked, hence $I(L, B|S)$.

3. (a) $A - T - E - L - S$ is blocked at L (head-to-tail) because $L \in \{L\}$.
   (b) $A - T - E - D - B - S$ is blocked at D (head-to-head) because $D \notin \{L\}$.
   All paths are blocked, hence $I(A, S|L)$.

4. $A - T - E - D - B - S$ is unblocked at
   (a) D (head-to-head) because $D \in \{L, D\}$;
   (b) T, E, B are all head-to-tail and are not in the evidence set.

   $A - T - E - D - B - S$ unblocked, hence $\neg I(A, S|L, D)$. Note that $A - T - E - L - S$ remains blocked.