PMR – α and β recursion for HMMs

Dec 2012

$$\alpha(z_{n+1}) = p(x_1, \dots x_n, x_{n+1}, z_{n+1})$$

$$= \sum_{z_n} p(x_1, \dots, x_n, x_{n+1}, z_n, z_{n+1})$$

$$= \sum_{z_n} p(x_{n+1} | x_1, \dots x_n, z_n, z_{n+1})\, p(x_1, \dots x_n, z_n, z_{n+1})$$

$$= \sum_{z_n} p(x_{n+1} | z_{n+1})\, p(z_{n+1} | z_n, x_1, \dots x_n)\, p(x_1, \dots x_n, z_n)$$

$$= \sum_{z_n} p(x_{n+1} | z_{n+1})\, p(z_{n+1} | z_n)\, p(x_1, \dots x_n, z_n)$$

$$= \sum_{z_n} \alpha(z_n)\, a_{z_n z_{n+1}}\, p(x_{n+1} | z_{n+1})$$

$$\beta(z_n) = p(x_{n+1}, \dots x_N | z_n)$$

$$= \sum_{z_{n+1}} p(x_{n+1}, \dots x_N, z_{n+1} | z_n)$$

$$= \sum_{z_{n+1}} p(x_{n+1} \dots x_N | z_{n+1}, z_n)\, p(z_{n+1} | z_n)$$

$$= \sum_{z_{n+1}} p(x_{n+1} \dots x_N | z_{n+1})\, p(z_{n+1} | z_n)$$

$$= \sum_{z_{n+1}} p(x_{n+1} | z_{n+1})\, p(x_{n+2}, \dots x_N | z_{n+1})\, p(z_{n+1} | z_n)$$

$$= \sum_{z_{n+1}} \beta(z_{n+1})\, a_{z_n z_{n+1}}\, p(x_{n+1} | z_{n+1})$$