

PMR - α and β recursions for HMMs

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$$\begin{aligned}\alpha(z_{n+1}) &= p(x_1, \dots, x_n, z_{n+1}) \\ &= \sum_{z_n} p(x_1, \dots, x_n, z_n, z_{n+1}) \\ &= \sum_{z_n} p(x_{n+1} | x_1, \dots, x_n, z_n, z_{n+1}) p(x_1, \dots, x_n, z_n, z_{n+1}) \\ &= \sum_{z_n} p(x_{n+1} | z_{n+1}) p(z_{n+1} | z_n, x_1, \dots, x_n) p(x_1, \dots, x_n, z_n) \\ &= \sum_{z_n} p(x_{n+1} | z_{n+1}) p(z_{n+1} | z_n) p(x_1, \dots, x_n, z_n) \\ &= \sum_{z_n} \alpha(z_n) a_{z_n, z_{n+1}} p(x_{n+1} | z_{n+1})\end{aligned}$$

$$\begin{aligned}\beta(z_n) &= p(x_{n+1}, \dots, x_N | z_n) \\ &= \sum_{z_{n+1}} p(x_{n+1}, \dots, x_N, z_{n+1} | z_n) \\ &= \sum_{z_{n+1}} p(x_{n+1}, \dots, x_N | z_{n+1}, z_n) p(z_{n+1} | z_n) \\ &= \sum_{z_{n+1}} p(x_{n+1}, \dots, x_N | z_{n+1}) p(z_{n+1} | z_n) \\ &= \sum_{z_{n+1}} p(x_{n+1} | z_{n+1}) p(x_{n+2}, \dots, x_N | z_{n+1}) p(z_{n+1} | z_n) \\ &= \sum_{z_{n+1}} \beta(z_{n+1}) a_{z_n, z_{n+1}} p(x_{n+1} | z_{n+1})\end{aligned}$$