3/11/10

PMR – EM for a Mixture of Gaussians

$$Q(\theta \mid \theta^{old}) = \sum_i Q_i(\theta \mid \theta^{old})$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{k} p(z_i = j \mid x_i, \theta^{old}) \log \left\{ \pi_j \, p(x_i \mid z_i = j, \theta_j) \right\}$$

for a Gaussian in 1-d

$$\log p(x_i \mid z_i = j, \theta_j) = -\frac{1}{2} \log (2\pi\sigma_j^2) - \frac{1}{2} \frac{(x_i - \mu_j)^2}{\sigma_j^2}$$

$\therefore$

$$\frac{\partial Q}{\partial \mu_l} = \sum_{i=1}^{n} p(z_i = l \mid x_i, \theta^{old}) \frac{(x_i - \mu_l)}{\sigma_l^2} = 0$$

$$\therefore \quad \mu_l \sum_{i=1}^{n} p(z_i = l \mid x_i, \theta^{old}) = \sum_{i=1}^{n} p(z_i = l \mid x_i, \theta^{old}) \, x_i$$

or

$$\mu_l^{new} = \frac{\sum_{i=1}^{n} p(z_i = l \mid x_i, \theta^{old}) \, x_i}{\sum_{i=1}^{n} p(z_i = l \mid x_i, \theta^{old})}$$

KL – divergence

using $\log z \leq z - 1$

$$\log \frac{Q_i}{P_i} \leq \frac{Q_i}{P_i} - 1$$

$$\therefore \quad \sum_i P_i \log \frac{Q_i}{P_i} \leq \sum_i [Q_i - P_i] = 0 \quad \text{as} \quad \sum_i Q_i = \sum_i P_i = 1$$

x by -1 to give

$$\sum_i P_i \log \frac{P_i}{Q_i} \geq 0$$