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PMF: CONDITIONAL DISTRIBUTIONFOR THE BOLTZMANN MACHINE

$$p(x_k | \text{rest}) = \frac{\exp\left(\frac{1}{2} \sum_{i,j} w_{ij} x_i x_j\right)}{\sum_{x_k = \pm 1} \exp\left(\frac{1}{2} \sum_{i,j} w_{ij} x_i x_j\right)}$$

everything not involving x_k cancels top and bottom

$$\text{Let } h_k = \sum_j w_{kj} x_j$$

Then

$$p(x_k | \text{rest}) = \frac{\exp(h_k x_k)}{\sum_{x_k = \pm 1} \exp(h_k x_k)}$$

so

$$p(x_k = 1 | \text{rest}) = \frac{e^{h_k}}{e^{h_k} + e^{-h_k}} = \frac{1}{1 + e^{-2h_k}} = \sigma(2h_k)$$