PMF: Worked Example of Junction Tree Alg.

\[ p(b|c) \]

\[
\begin{array}{c|cc}
  c & b=0 & b=1 \\
  \hline
  c=0 & 0.1 & 0.9 \\
  c=1 & 0.8 & 0.2 \\
\end{array}
\]

\[ p(c) \]

\[
\begin{array}{c|cc}
  c & 0.8 & 0.2 \\
  \hline
  c=0 & 0.8 \\
  c=1 & 0.2 \\
\end{array}
\]

\[ p(a|b) \]

\[
\begin{array}{c|cc}
  a & b=0 & b=1 \\
  \hline
  a=0 & 0.6 & 0.3 \\
  a=1 & 0.4 & 0.7 \\
\end{array}
\]

\[ \gamma(a, b) = p(a|b) \quad \gamma(b, c) = p(c) p(b|c) \]

\[ \gamma(b, c) = \]

\[
\begin{array}{c|cc}
  c & b=0 & b=1 \\
  \hline
  c=0 & 0.08 & 0.72 \\
  c=1 & 0.16 & 0.04 \\
\end{array}
\]

\[ \phi(b) \text{ is initialized to } (1, 1) \]

\[ \phi^*(b) = \sum_a \gamma(a, b) = (1, 1) \]

\[ \phi^*(b, c) = \gamma(b, c) \frac{\phi^*(b)}{\phi(b)} = \gamma(b, c) = p(b, c) \]

\[ p(b) = \sum_c p(b, c) = (0.08 + 0.16, 0.72 + 0.04) \]

\[ = (0.24, 0.76) \]

\[ b=0 \quad b=1 \]
Now consider $P(b | a = 0, c = 1)$

<table>
<thead>
<tr>
<th>$b$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi(b, c = 1)$</td>
<td>0.16</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Slice of $\Psi(b, c)$ Table from previous page

$\phi(b)$ is multiplied to (1, 1)

$\Psi(a = 0, b)$

<table>
<thead>
<tr>
<th>$b$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

Slice of $\Psi(a, b)$ Table from previous page

Passing from $\mathbb{E}B$ to $\mathbb{B}n$

$\phi(b) = (0.6, 0.3)$

No summation, as the table is sliced

$\Psi^*(b, c = 1) = \Psi(b, c = 1) \cdot \phi(b) = (0.16 \times 0.6, 0.04 \times 0.3)

\phi(b) = (0.096, 0.012)$

At convergence $\Psi^*(b, c = 1) = P(b, a = 0, c = 1)$

$P(b = 0 | a = 0, c = 1) = \frac{0.096}{0.096 + 0.012} = 0.889$

$P(b = 1 | a = 0, c = 1) = \frac{0.012}{0.096 + 0.012} = 0.111$