PMR: HOLMES/WATSON EXAMPLE

$p(r = yes) = 0.2$

$p(s = yes) = 0.1$

$p(w = y \mid r = y) = 1$

$p(w = y \mid r = n) = 0.2$

$p(h = y \mid r = y, s = y) = 1.0$

$p(h = y \mid r = y, s = n) = 1.0$

$p(h = y \mid r = n, s = y) = 0.9$

$p(h = y \mid r = n, s = n) = 0.0$

Holmes = y evidence

$$p(r, s \mid h = y) = \frac{p(r, s, h = y)}{p(h = y)}$$

$$p(r, s, h) = \sum_w p(r, s, h, w) = p(r)\, p(s)\, p(h \mid r, s) \sum_w p(w \mid r)$$

$$= p(r)\, p(s)\, p(h \mid r, s)$$

$p(r = y, s = y, h = y) = 0.2 \times 0.1 \times 1.0 = 0.02$

$p(r = y, s = n, h = y) = 0.2 \times 0.9 \times 1.0 = 0.18$

$p(r = n, s = y, h = y) = 0.8 \times 0.1 \times 0.9 = 0.072$

$p(r = n, s = n, h = y) = 0.8 \times 0.9 \times 0.0 = 0$

$p(h = y) = 0.272$

$\therefore p(r = y, s = y \mid h = y) = \frac{0.02}{0.272} = 0.074$

$p(r = y, s = n \mid h = y) = \frac{0.18}{0.272} = 0.662$

$$p(r=n, s=y \mid h=y) = \frac{0.072}{0.272} = 0.264$$

$$p(r=n, s=n \mid h=y) = \frac{0}{0.272} = 0.0$$

Marginals

$$p(r=y \mid h=y) = \sum_s p(r=y, s \mid h=y) = 0.074 + 0.662 = 0.736$$

$$p(s=y \mid h=y) = \sum_r p(r, s=y \mid h=y) = 0.074 + 0.264 = 0.338$$

Note also that

$$p(r=y, s=y \mid h=y) \neq p(r=y \mid h=y)\, p(s=y \mid h=y)$$

$$0.074 \neq 0.736 \times 0.338 = 0.249$$

i.e. r and s are conditionally dependent