Probabilistic Modelling and Reasoning Course Note. Component 2: Factor Graphs.

The first thing you need to do for this is ensure you have done a thorough review of your graph theory. David Barber covers the graph theory you need in Chapter 2.

This component introduces the idea of a factor graph. In outline, we

1. Note that we model things with probability distributions.
2. Note that these distributions can usefully be represented as a product of a number of factors.
3. Each factor in this product may often involve only a few of the variables.
4. The allocation of variables to factors can be represented as a bipartite graph called a factor graph.

You should review the factor graph lecture notes. The following resources may also help:

https://people.kth.se/~tjtkoski/factorgraphs.pdf

http://www.psi.toronto.edu/~psi/pubs2/1999%20and%20before/134.pdf

http://web.cs.iastate.edu/~honavar/factorgraphs.pdf

But note that most resources on factor graph resources assume lots of other graphical model knowledge first. It is probably worth returning to factor graphs and reviewing many other resources at a later stage.

As you review these, you should be able

- To think of examples of real situations that you could represent using factored representations in probabilistic models.
- To start with a factored probability distribution and build the corresponding factor graph.

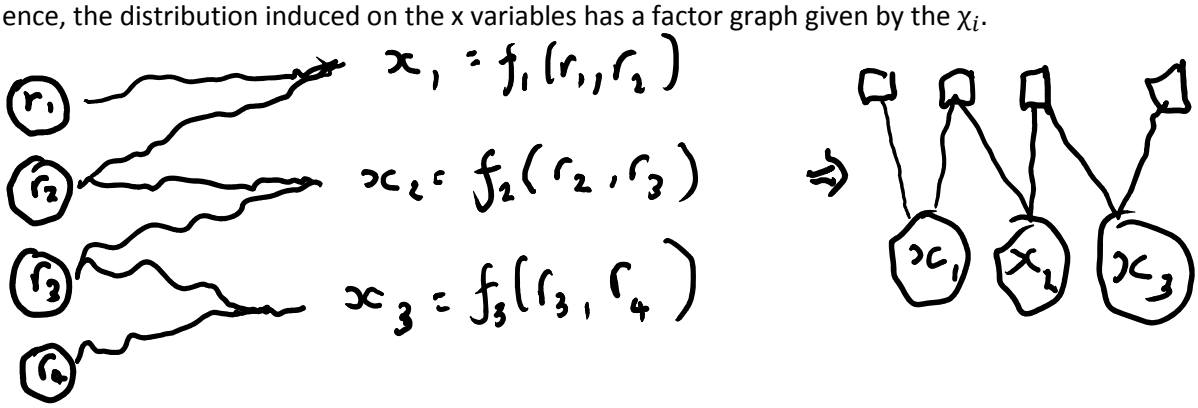
Factor graphs from factors

In the lectures, we saw one view of factor graphs – as a property of a distribution. This makes the most sense and is the most general way to view factor graphs, but for some it is not the most intuitive.

Here we outline an alternative viewpoint, based on the idea of generating distributions.

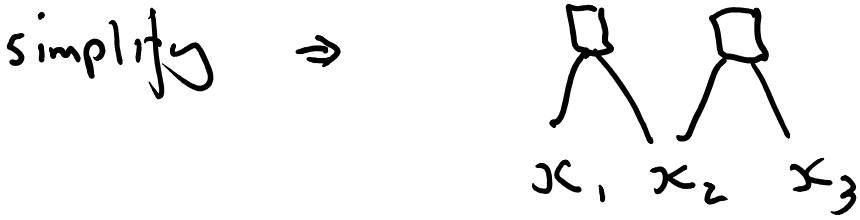
Consider a generative model of the following form.

First we have k independent random variables labelled r. Then we have m variables labelled x. Now each of the x_i are functions of a subset χ_i of the r variables. Now because of the deterministic dependence, the distribution induced on the x variables has a factor graph given by the χ_i.



Simplification

Undirected factor graphs can often and easily be simplified. If nodes connected to any factor is a subset of any other factor that first factor can be removed. Directed factors cannot be removed.



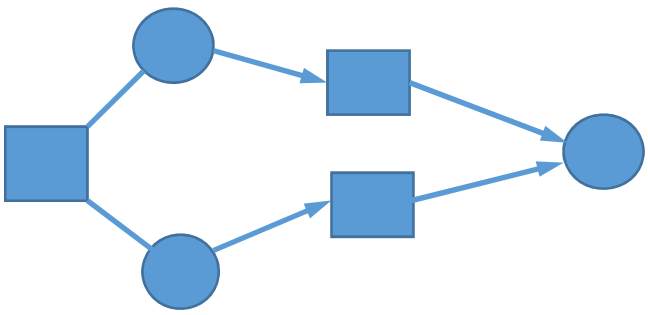
Further work

It is vital to work through the ideas behind graphical models and factor graphs in David Barber's book.

Exam-Style Questions.

1) Draw a factor graph for the distribution

$$P(x_1, x_2, x_3, x_4, x_5) = K\ (x_1 + x_2 + x_3)^2 \exp(x_2 + x_5 + x_3 x_4 x_5)$$

Where K is a constant, and x_i is binary.

2) Draw the factor graph for the distribution $P(x_1, x_2, x_4, x_5 | x_3)$ obtained from the distribution in Q1.
3) Draw the directed factor graph for a distribution of the form $P(x_1|x_5, x_4)P(x_3|x_4)P(x_4)P(x_5)$. Can you draw an undirected factor graph that is true for the same distribution?
4) Can you add an edge to a factor graph and make it an invalid factor graph for the corresponding distribution?
5) Given a distribution, and a factor graph for that distribution, are the factor potentials (the functions associated with each factor) unique in general? If not give an example.
6) Does any undirected factor graph correspond to some valid distribution? Does any directed factor graph correspond to some valid distribution? If not, give a counter-example.
7) HARD: Give a valid form of distribution corresponding to the mixed factor graph: Is a graph without the undirected factor possible?