# A Worked Example of Probabilistic Inference Using the Junction Tree Algorithm

Probabilistic Modelling and Reasoning
Instructor: Dr Chris Williams
School of Informatics, University of Edinburgh

September 2005

Consider the belief network shown in Figure 1 which has eight nodes. These nodes take on the values:

| | |
|---|---|
| $L$ | $\{peace,\ war\}$ |
| $Q$ | $\{peace,\ war\}$ |
| $F$ | $\{yes,\ no\}$ |
| $B$ | $\{run,\ stay\}$ |
| $G$ | $\{\uparrow,\ 0,\ \downarrow\}$ |
| $S$ | $\{yes,\ no\}$ |
| $H$ | $\{yes,\ no\}$ |
| $I$ | $\{yes,\ no\}$ |

The story behind this network is given in the Appendix.

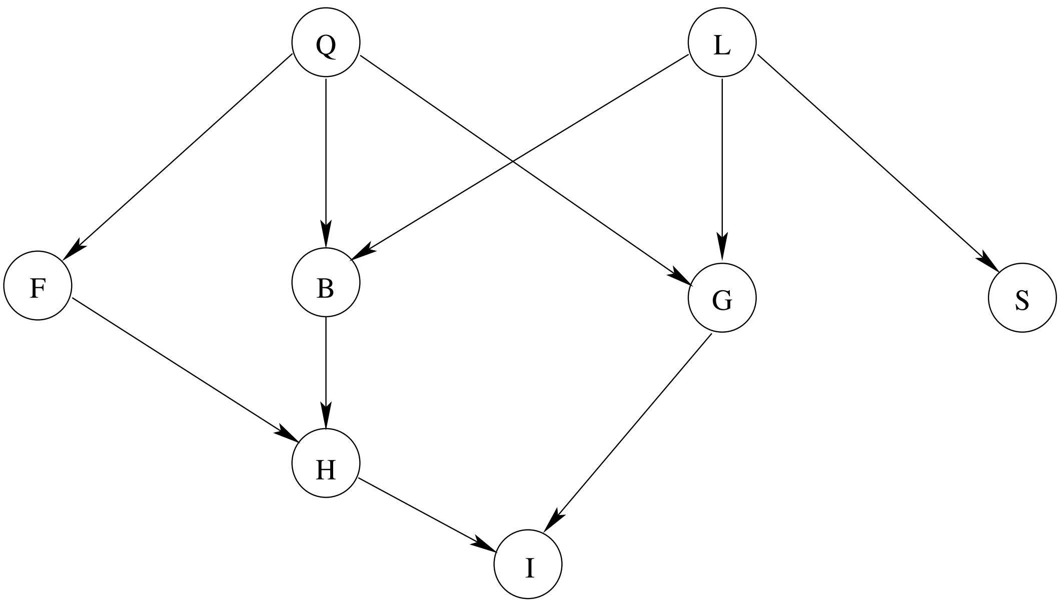


Figure 1: Structure of the Belief Network

The CPTs for the network are given as follows (where applicable, $\uparrow$, $\downarrow$, and 0 stands for increase, decrease, and no change respectively):

$$\begin{array}{rl}
P(L = peace) = 0.4 & P(Q = peace) = 0.6 \\
P(S = yes|L = peace) = 0.8 & P(S = yes|L = war) = 0.3 \\
P(F = yes|Q = peace) = 0.8 & P(F = yes|Q = war) = 0.1 \\
P(B = run|L = peace, Q = peace) = 0.2 & P(B = run|L = peace, Q = war) = 1 \\
P(B = run|L = war, Q = peace) = 1 & P(B = run|L = war, Q = war) = 1 \\
P(G =\uparrow |L = peace, Q = peace) = 0.3 & P(G = 0|L = peace, Q = peace) = 0.6 \\
P(G =\uparrow |L = war, Q = peace) = 0.1 & P(G = 0|L = war, Q = peace) = 0.2 \\
P(G =\uparrow |L = peace, Q = war) = 0.8 & P(G = 0|L = peace, Q = war) = 0.1 \\
P(G =\uparrow |L = war, Q = war) = 0.2 & P(G = 0|L = war, Q = war) = 0.2 \\
P(H = yes|B = run, F = no) = 1 & P(H = yes|B = run, F = yes) = 0.4 \\
P(H = yes|B = stay, F = no) = 0.5 & P(H = yes|B = stay, F = yes) = 0.1 \\
P(I = yes|G = 0, H = yes) = 0 & P(I = yes|G = 0, H = no) = 0.3 \\
P(I = yes|G =\downarrow, H = yes) = 0 & P(I = yes|G =\downarrow, H = no) = 0.1 \\
P(I = yes|G =\uparrow, H = yes) = 0 & P(I = yes|G =\uparrow, H = no) = 1
\end{array}$$

One of the possible junction tree for the network is given by

```
                                {F Q B H}
                                    |
{S L} --- {L Q B G} --- {H B Q G} --- {I G H}
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This junction tree can be obtained using the elimination ordering $S,\ I,\ L,\ F,\ Q,\ G,\ B,\ H$. Use the assignment of potentials $\psi(SL) = P(L)P(S|L)$, $\psi(LQBG) = P(B|L,Q)P(G|L,Q)$, $\psi(HBQG) = p(Q)$, $\psi(FQBH) = P(F|Q)P(H|B,F)$, $\psi(IGH) = P(I|H,G)$.

**The question we wish to answer is: Using the junction tree algorithm calculate** $p(H = yes|Q = war, G = 0, F = yes)$**.**

The separators are initialized thus: $\phi(L) = \phi(BHQ) = \phi(QBG) = \phi(GH) = 1$.

An appropriate choice of the root node is HBQG although other nodes are suitable as long as they contain the query variable H. We first collect all the evidence in the separators GH, BHQ and QBG and then update HBQG in only one step.

**Absorption from IGH to GH**
$\phi^*(GH) = \sum_I P(I|H, G = 0) = 1$

**Absorption from FQBH to BHQ**
$\phi^*(BHQ) = \sum_F \psi(FQBH)\delta(F = yes, Q = war) = P(F = yes|Q = war)P(H|B, F = yes)$. Note that the sum over F is not done as F is evidential.

**Absorption from SL to LQBG and from LQBG to QBG**

$$\begin{aligned}\psi^*(LQBG) &= \frac{\sum_s \psi(SL)}{\phi(L)}\psi(LQBG)\delta(Q = war, G = 0) \\ &= \left(\sum_s P(L)P(S|L)\right) P(B|L, Q = war)P(G = 0|L, Q = war) \\ &= P(L)P(B|L, Q = war)P(G = 0|L, Q = war).\end{aligned}$$

$\phi^*(QBG) = \sum_L \psi^*(LQBG) = \sum_L P(L)P(B|L, Q = war)P(G = 0|L, Q = war)$

**Update of HBQG**

$$\psi^*(HBQG) = \frac{\phi^*(GH)\phi^*(BHQ)\phi^*(QBG)}{\phi(GH)\phi(BHQ)\phi(QBG)}\psi(HBQG)\delta(Q = war, G = 0)$$

$$\psi^*(HBQG) = P(Q = war)P(F = yes|Q = war)P(H|B, F = yes)\times \\ \sum_L P(L)P(B|L, Q = war)P(G = 0|L, Q = war).$$

One can also do the update of HBQG in three stages, incorporating the three messages one-at-a-time. Now we have collected so the evidence to the node HBQG. The distribution of the evidence is not necessary as this node will not be further updated and it contains the required probability. More specifically:

$$\psi^*(HBQG) = P(H, B, F = yes, G = 0, Q = war)$$

Therefore, we can calculate $P(H = yes|Q = war, G = 0, F = yes)$ as follows:

$$P(H = yes|Q = war, G = 0, F = yes) = \frac{\sum_B \psi^*(HBQG)\delta(H = yes)}{\sum_{H,B} \psi^*(HBQG)} \tag{1}$$

Replacing the values obtained above in Equation 1 we easily find that $P(Q = war)$ and $P(F = yes|Q = war)$ cancel from the equation as they do NOT depend on B nor H and are the same for the numerator and denominator.

Additionally, let us denote:

$$m_L(B) = \sum_L P(L)P(B|L, Q = war)P(G = 0|L, Q = war)$$

Thus, Equation 1 can be rewritten as:

$$P(H = yes|Q = war, G = 0, F = yes) = \frac{\sum_B m_L(B)P(H = yes|B, F = yes)}{\sum_B m_L(B)\sum_H P(H|B, F = yes)}$$

$$= \frac{m_L(B = run)P(H = yes|B = run, F = yes) + m_L(B = stay)P(H = yes|B = stay, F = yes)}{m_L(B = run) + m_L(B = stay)}$$

A quick look at the CPTs allows us to determine that $m_L(B = stay) = 0$ as $P(B = stay|L, Q = war) = 0$ from which the answer follows:

$$P(H = yes|Q = war, G = 0, F = yes) = P(H = yes|B = run, F = yes) = 0.4$$

## Appendix: The Story of the Quickfeet and the Longarms

The belief network corresponds to the following imaginary scenario: You are the chief of a hunting clan, the *Quickfeet* [**Q**]. You have a long-time dispute with your traditional opponents, the *Longarms* [**L**] over goat skins [**G**], which both clans use for making clothes. Both you and your opponents may try to resolve the dispute by choosing either negotiating, or taking military actions. You anticipate that if you go for a peaceful strategy, while they go for a military solution, you are not very likely to gain any skins and may lose many of your own, and vice versa. If both of you start fighting, it is likely that some of the skins will be damaged. If both you and your opponents go for peace, it is most likely that the number of the skins will not change much. You also know that you the *Quickfeet* are a little more successful in negotiations, while the *Longarms* are a little more successful in fighting. You are also traditionally more peaceful than your opponents. Life is uncertain, and no matter what the *Longarms*' and the *Quickfeet*'s strategies are, there is a chance of losing some skins to the weather and moths, and gaining some by hunting.

You do not know what your opponents will do. One clue you may use to decide which strategy they have chosen is the smoke rising up the hills. The smoke [**S**] may indicate that your opponents are smoking the peace pipes, which may happen once they have decided to negotiate. On the other hand, there is a small chance that they will be burning heather, which the *Longarms* sometimes do before starting a war.

Your clan possesses the skill of growing crops and vegetables. Whether or not you have enough time for farming [**F**] depends on whether you go for war or peace with the *Longarms*. Hunting beasts [**B**] and having time for farming is essential for avoiding hunger [**H**] in the approaching winter. You know that if you decide to fight, the beasts are likely to get scared and run away.

Finally, the improvement [**I**] of your clan's life quality depends on whether you can avoid hunger and get enough goat skins for making clothes.