Performance Modelling

Practical 1

28th September 2017

This is the first of two practicals. The coursework accounts for 25% of the marks for the course, and this practical accounts for 50% of the coursework mark. Each practical is marked out of 50. The deadline for the first practical is 16:00 on **Wednesday 25th October**. This is an individual practical: whilst general discussion with classmates is encouraged, the solutions that you submit must be your own work.

This practical is intended to give you some hands-on experience of using the operational laws, Markov processes and queueing networks. The second practical will involve larger modelling studies using GSPNs and PEPA.

Good Scholarly Practice: Please remember the University requirement as regards all assessed work for credit. Details about this can be found at:

`http://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct`

Furthermore, you are required to take reasonable measures to protect your assessed work from unauthorised access. For example, if you put any such work on a public repository then you must set access permissions appropriately (generally permitting access only to yourself, or your group in the case of group practicals).

1 Operational Laws

A social media web site has an architecture consisting of a web server, an application server and a database server. The database server has two processors and three disks. The application server converts updates and queries from the users — "*social interactions*" — from the web server into transactions that are submitted to the database.

On average, each user visit to the web site generates 8 consecutive social interactions and each social interaction generates 2 database transactions. An average transaction generates 4 accesses to disk A, 3 accesses to disk B, 2 accesses to disk C and a total of 10 CPU bursts which are evenly distributed across the two processors.

Average service times per access at the three disks are 50, 60 and 25 milliseconds respectively, while average service times at the database processors are 10 milliseconds per burst. The average service demand of each social interaction is 120 milliseconds at the web server and 150 milliseconds at the application server. On average, a user thinks for 30 seconds between each social interaction.

For all parts you should state which operational laws you are using and explain your reasoning.

a.) *Which is the bottleneck device in the database?*

2 marks

b.) *What is the minimum average residence time per transaction, regardless of the number of users using the system?*

3 marks

c.) *What is the maximum possible utilisation of disk C and when would this be achieved?*

3 marks

d.) *How many users would be using the system if the average response time per social interaction is 12 seconds and the throughput at disk C is 0.02 (accesses/millisecond)?*

3 marks

e.) *What revised service time per access at disk A would you recommend to achieve a system response time per social interaction of 6 seconds when there are 180 users on the system? Which other devices, if any, would need to be speeded up to achieve this target?*

4 marks

15 marks total

2 Markov Processes

The owner of a small internet cafe currently has five PCs based on a single wireless network with a broadband connection to the Internet. She has observed that at peak times customers are leaving because they cannot get access to a PC and she wonders if she could increase her profits. Preliminary studies have shown that the average customer occupies a PC for 30 minutes and currently at peak times 15 customers arrive per hour. Customers who arrive to find all the PCs occupied immediately leave and go elsewhere. You are asked to construct a Markov process model to investigate her problem.

a.) *What random variable would you use to characterise the state of the system? Justify your answer.*

2 marks

b.) *Draw the state transition diagram for the Markov process.*

3 marks

c.) *Give the infinitesimal generator matrix for this Markov process.*

2 marks

d.) *Using* `MatLab` *or otherwise, solve the global balance equations for the process and give the steady state probability distribution.*

3 marks

e.) *What is the loss rate, i.e. the rate that customers leave the system without gaining access to a PC? Explain how you arrived at your answer.*

2 marks

f.) *The cafe owner considers two alternatives*

  (a) *Buying one additional PC with the same specification as the existing PCs;*
  (b) *Upgrading each of the existing PCs and the comms so that the average user takes 20 minutes instead of 30 minutes.*

Construct a modification of your model for each of these alternatives and use these new models to determine which is the better alternative in terms of loss rate.

6 marks

g.) *The cafe owner knows that on average each customer spends 10p on drinks and snacks per minute that they spend in the cafe. The existing PCs cost 2p per minute in energy consumption, whereas the upgraded machines cost 3p per minute in energy consumption.*

(a) *Which of the two options above will bring in the most revenue on drinks and snacks?*

(b) *Compare the energy costs of the two options above.*

(c) *If you were the cafe owner, which option would you go for? Justify your answer.*

6 marks

24 marks total

3 Modelling with Queues

For each of the following scenarios, using Kendall's notation, identify which type of queue you believe would be most appropriate to represent it. In each case you should justify your answer. There will be marks for the justification as well as for the classification.

a.) *In a small coffee shop there is typically only one person serving behind the counter, taking the money and making the coffee for each customer, taking an amount of time which is exponentially distributed. Customers, who arrive according to a Poisson stream, wait in a single queue until they are served. On saturday mornings the queue sometimes reaches out of the door and onto the street outside.*

2 marks

b.) *At an airport check-in, in the two hours before a flight passengers arrive in a Poisson stream. The time taken to check a passenger in to their flight is highly variable, depending on whether they have luggage or not, and connecting flights or not. There are typically two members of ground staff working on the check-in desk,and, although the shared queue can grow very long, all passengers are checked in eventually.*

2 marks

c.) *Cars arrive at a petrol station at random times during the day; the inter-arrival times have been observed to be normally distributed. There are 8 pumps at the petrol station and space for 6 more cars waiting; cars arriving when all these places are taken will drive on to another petrol station. Whilst all pumps work at the same rate the time taken also depends on the capacity of the tank of the car.*

2 marks

d.) *Patients arrive at the accident and emergency department of a busy hospital throughout the night; an exponential distribution gives a reasonable fit to the inter-arrival times. There is significant variation in the time that patients need in treatment but measurement suggests that it follows a log normal distribution. Most patients are seen on a first come first served basis but when a severe case is admitted it goes straight to the head of the queue.*

2 marks

e.) *The city car club currently has 1000 members and 250 cars. When a member needs a car he/she can take the nearest available car and later returns it at the end of his/her journey; journey times have been found to be normally distributed. If a car is not currently available the member waits at their chosen location until a car arrives. For each member the time between journeys is exponentially distributed.*

3 marks

11 marks total

You should submit your solution on paper to the ITO before the deadline. Hand-drawn diagrams are fine but other answers should be typed.

The deadline for the homework is 16:00 on **WEDNESDAY 25th October 2017**. There will be a penalty for late submission. See
`http://web.inf.ed.ac.uk/infweb/student-services/ito/admin/coursework-projects/late-coursework-extension-requests`
for the school late policy.

Marked work will be returned on or before 8th November 2017.