Performance Modelling (Inf4/M.Sc.)

Practical 1

27th September, 2013

*This is the first of two practicals. The coursework accounts for 25% of the marks for the course, and this practical accounts for 50% of the coursework mark. Each practical is marked out of 50. The deadline for the first practical is 16:00 on **Thursday 27th October 2013**. This is an individual practical: whilst general discussion with classmates is fine, the solutions that you submit must be your own work. The School policy on academic conduct can be found at*
http://www.ed.ac.uk/schools-departments/academic-services/students/postgraduate-taught/
discipline/academic-misconduct

This practical is intended to give you some hands-on experience of using the operational laws, Markov processes and queueing networks. The second practical will involve larger modelling studies using GSPNs and PEPA.

1 Operational Laws

A Web-based brokerage company runs a site consisting of a web server, an application server and a database server. The database server has two processors and three disks. The application server converts business functions from the web server into transactions that are submitted to the database.

On average, each user visit to the web site generates 4 consecutive business functions and each business function generates 1.5 database transactions. An average transaction generates 3 accesses to disk A, 4 accesses to disk B, 2 accesses to disk C and a total of 10 CPU bursts which are evenly distributed across the two processors.

Average service times per access at the three disks are 50, 60 and 30 milliseconds respectively, while average service times at the database processors are 10 milliseconds per burst. The average service demand of each business function is 160 milliseconds at the web server and 190 milliseconds at the application server. On average, a user thinks for 30 seconds between each business function.

For all parts you should state which operational laws you are using and explain your reasoning.

a.) *Which is the bottleneck device in the database?*

2 marks

b.) *What is the minimum average residence time per transaction, regardless of the number of users using the system?*

3 marks

c.) *What is the maximum possible utilisation of disk C and when would this be achieved?*

3 marks

d.) *How many users would be using the system if the average response time per business function is 12 seconds and the throughput at disk C is 0.005 (accesses/millisecond)?*

3 marks

e.) *What revised service time per access at disk A would you recommend to achieve a system response time per business function of 10 seconds when there are 120 users on the system? Which other devices, if any, would need to be speeded up to achieve this target?*

4 marks

15 marks total

2   Loss probabilities within a social media system

In a customer account of a social media site, if a message arrives when the customer's unread message buffer is already full (all places occupied) a message must be discarded. The customer can choose settings to decide whether it is the oldest message or the newly arrived message that is discarded. But in either case it is clearly unsatisfactory that messages are being lost.

Construct a Markov process of an unread message buffer within the social media site that has 4 places. Messages arrive at the buffer at a rate of 3 messages per hour. The customer logs on to the system on average twice an hour. Once they have logged on they typically read all the messages present in the buffer until it is empty; they then typically log off in one minute if the incoming message buffer remains empty. On average a user takes three minutes to enjoy each message before moving on to the next.

You may assume that all times are exponentially distributed.

a.) *What random variables would you use to characterise the state of the system?*

2 marks

b.) *Draw the state transition diagram for the Markov process.*

4 marks

c.) *Give the infinitesimal generator matrix for this Markov process.*

3 marks

d.) *Using* `xmaple` *or otherwise, solve the global balance equations for the process and give the steady state probability distribution.*

3 marks

e.) *What is the loss rate, i.e. the rate at which messages are dropped without being read by a customer? Explain how you arrived at your answer.*

3 marks

f.) How will the loss rate improve if the buffer capacity is increased to five places? Construct a modification of your model for this alternative and use this new model to determine the percentage reduction in loss rate.

10 marks

25 marks total

3 Modelling with Queues

For each of the following scenarios, using Kendall's notation, identify which type of queue you believe would be most appropriate to represent it. In each case you should justify your answer.

a.) Cars arrive at a car wash in a Poisson stream. The car wash is suitable for all cars but only offers one wash programme which takes a fixed time of three minutes. Cars which arrive to find the car wash occupied wait in line.

2 marks

b.) Cars arrive at a petrol station at random times during the day; the inter-arrival times have been observed to be normally distributed. There are 6 pumps at the petrol station and space for 4 more cars waiting; cars arriving when all these places are taken will drive on to another petrol station. Whilst all pumps work at the same rate the time taken also depends on the capacity of the tank of the car.

2 marks

c.) Patients arrive at the accident and emergency department of a busy hospital throughout the night; an exponential distribution gives a reasonable fit to the inter-arrival times. There is significant variation in the time that patients need in treatment but measurement over several nights suggests that it follows a log normal distribution. Most patients are seen on a first come first served basis but when a severe case is admitted it goes straight to the head of the queue.

2 marks

d.) In the post office there are typically four counters open and each counter can deal with any customer, taking an amount of time which is exponentially distributed. Customers, who arrive according to a Poisson stream, wait in a common queue and when they reach the head of the queue they go to which ever counter is available next. On saturday mornings the queue sometimes reaches out of the door and onto the street outside.

2 marks

e.) The city car club currently has 100 members and 12 cars. When a member needs a car he/she can take the nearest available car and later returns it at the end of his/her journey; journey times have been found to be normally distributed. If a car is not currently available the member waits at their chosen location until a car arrives. For each member the time between journeys is exponentially distributed.

2 marks

10 marks total

You should submit your solution on paper to the ITO before the deadline. Hand-drawn diagrams are fine but other answers should be typed.

The deadline for the homework is 16:00 on **THURSDAY 24th OCTOBER 2013**. *There will be a penalty for late submission. See*
`http://www.inf.ed.ac.uk/student-services/teaching-organisation/for-taught-students/` `coursework-and-projects/late-coursework-submission` *for the school late policy.*

Marked work will be returned on or before 8th November 2013.