Performance Modelling — Traffic Equation Exercise

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1. Write down the traffic equations for the network.

2. If the value of $\lambda$ is 9, $p = 0.2$ and $q = 0.5$, what is the effective arrival rate at point $A$?

3. Using the same values, what is the rate of external departures at point $B$ in the network? Explain your reasoning.

4. If the service rate at service centre 1 is $\mu_1 = 20$, what is the probability that this queue is empty but the server is not idle? Explain your reasoning.
The traffic equations for the network are:

\[ \lambda_1 = \lambda + (1 - q)\lambda_2 \]  
\[ \lambda_2 = p\lambda_1 \]  
\[ \lambda_3 = (1 - p)\lambda_1 \]
Exercise

The traffic equations for the network are:

\[ \lambda_1 = \lambda + (1 - q)\lambda_2 \]  \hspace{1cm} (1)
\[ \lambda_2 = p\lambda_1 \]  \hspace{1cm} (2)
\[ \lambda_3 = (1 - p)\lambda_1 \]  \hspace{1cm} (3)

To find the effective arrival rate at A we need to substitute (2) into (1)

\[ \lambda_1 = \lambda/(1 - p + pq) \]

and substitute in the values giving \( \lambda_1 = 9/0.9 = 10 \)
So the effective arrival rate at point A is 10.
The external departures at point $B$ are all the departures from service centre 3 and this will have the same rate as the effective arrival rate, $\lambda_3$. From (3) $\lambda_3 = (1 - p)\lambda_1 = 0.8 \times 10 = 8$.

So external departures at point $B$ have rate 8.
Service rate at service centre 1 is $\mu_1 = 20$, implies $\rho_1 = 10/20 = 1/2$. This is a product form network so the queues behave as if independent i.e. we consider service centre 1 alone. "The queue is empty but the server is not idle" is true iff there is exactly one customer in the queue, i.e. $\pi_1(1)$. This is an $M/M/1$ queue so this can be expressed as $\rho(1 - \rho)$. Substituting in the value for $\rho_1$ we get 0.25.