Performance Modelling — Lecture 5
Queueing Networks

Jane Hillston
School of Informatics
The University of Edinburgh
Scotland

30th January 2017

Single Queues

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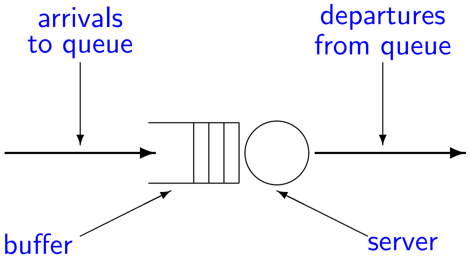
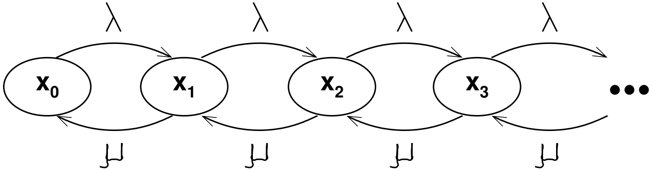
- The service facility has one or more servers who are capable of performing the service required by customers.
- If a customer cannot gain access to a server it must join a queue, in a buffer, until a server is available.
- When service is complete the customer departs, and the server selects the next customer from the buffer according to the service discipline.

Simple Example



State space of a simple queue



Arrival Pattern of Customers

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We will primarily be considering queues in which the times between arrivals are assumed to be exponentially distributed.

However you should be aware that many other possibilities exist and are studied, such as bulk or batch arrivals and train arrivals.

Service Time Distribution

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If the average duration of a service interaction between a server and a customer is $1/\mu$ then μ is the service rate.

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It is assumed that the servers are indistinguishable from each other, so that it does not matter which server actually serves a customer—the results, and the service characteristics, will be the same.

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- an infinite server: there are always at least as many servers as there are customers, so that each customer can have a dedicated server as soon as it arrives in the facility. There is no queueing, (and no buffer) in such facilities.

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If there are more than c customers, the additional customers will have to wait in the buffer.

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If the buffer capacity is so large that it never affects the behaviour of the customers it is assumed to be infinite.

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When the size of the population is so large that there is no perceptible impact on the arrival process we assume that the population is infinite.

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Identifying different classes is a workload characterisation task.

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The commonly used service disciplines are

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We will only consider solution of systems with the FCFS service discipline.

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- SD denotes the service discipline, FCFS by default.

Example

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Making exponential assumptions about the arrival rate and the service rate we would model the gateway as an $M/M/1/13/\infty/FCFS$ queue with $\lambda = 0.125$ and $\mu = 0.5$ (/ms).

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These are combined into a single parameter which characterises a single or multiple server system, the traffic intensity.

Traffic intensity

$$\rho = \frac{\lambda}{c \times \mu}$$

Traffic intensity and stability

For the system to be stable, ρ must be less than 1: that is the arrival rate of customers must be less than the rate at which they are served.

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The alternative, if the arrival rate is greater than the service rate, would result in a queue which grew unboundedly.

Network of queues

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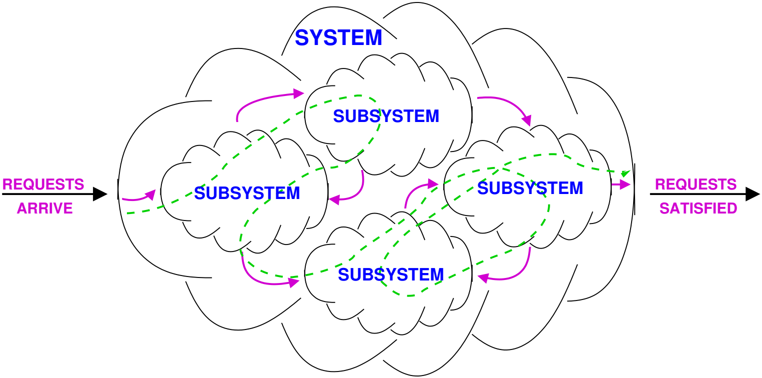

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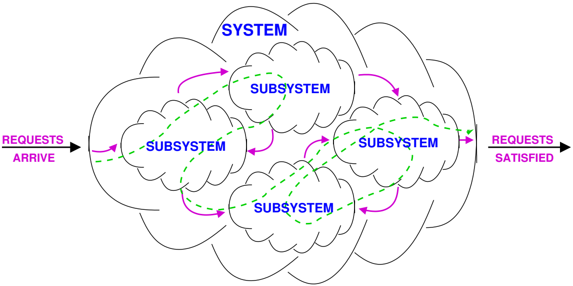

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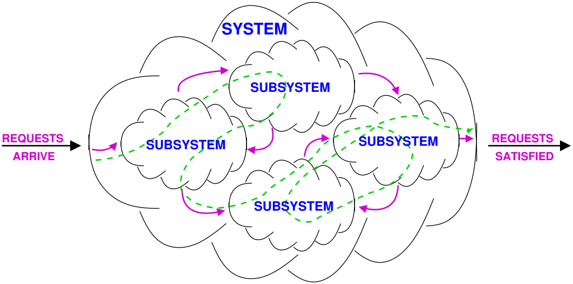

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After service at one service centre a customer may progress to other service centres, following some previously defined pattern of behaviour, corresponding to the customer's requirement.

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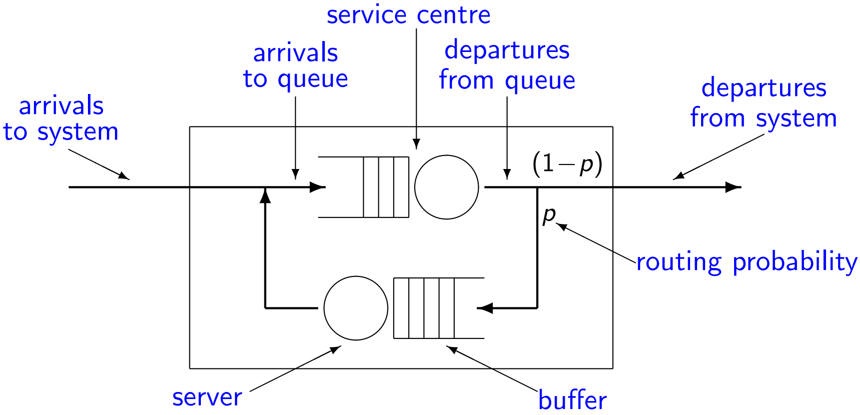

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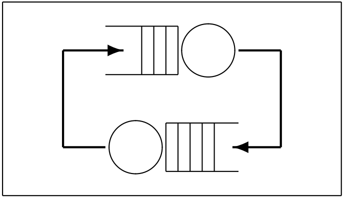

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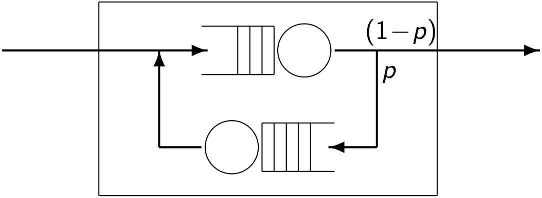

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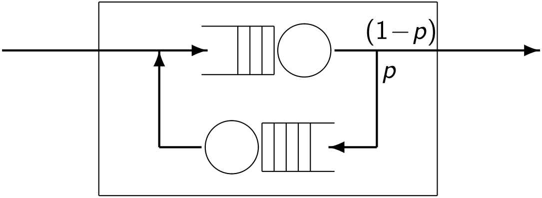

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- mixed, there are classes of customers within the system exhibiting open and closed patterns of behaviour respectively.

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If there a number of different classes of customers, the state is the number of customers of each class at each service centre.

As well as differences in arrival rate and/or service demand, classes may be distinguished by the routes which customers take through the network.

Expressiveness

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Fork and Join primitives used in computer systems to create and synchronise subprocesses, cause the number of jobs in the system to change and invalidate the assumption of independence among jobs.

Expressiveness cont.

Bulk and train arrivals. In communication networks the arrival of packets may occur in batches (bulk arrivals) or in quick succession (train arrivals). This breaks assumptions of independence between customers, and exponential inter-arrival times.

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Load dependent arrivals. Computer networks and distributed systems have intelligent load-balancing policies that cause new jobs or packets to go to one of a number of devices depending upon the load observed in the recent past.

Defections from the queue. Routers often set a maximum limit on the time that a packet or request is able to stay in a queue. Thus, packets may be dropped from the queue on the assumption that they may have already been retransmitted by higher protocol layers.