Performance Modelling — Lecture 14: Random Variables and Simulation

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9th March 2017

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  2. when a choice must be made within the behaviour of an entity we will sometimes want the decision to be made probabilistically.
- Both cases will involve sampling a probability distribution to extract a value each time this part of the entity's behaviour is reached.
- Both cases rely on the random number generator.

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- In general, each run of the simulation model provides a single estimate for these random variables.

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- We need more than a single estimate in order to draw conclusions about the system.
- We use output analysis techniques to improve the quality of an estimates and to develop ways of gaining more observations without excessive computational cost.
- Realistic simulation models take a long time to run—there is always a trade-off between accuracy of estimates and execution time.

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  2. The sequence is transformed to produce a sequence of random values which satisfy the desired distribution.
- This second step is called random variate generation.

Generating uniform random numbers

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- The sequence $X_k/(M-1)$ will then be approximately uniformly distributed over $(0, 1)$.
- In 1951, D.H. Lehmer discovered that the residues of successive powers of a number have good randomness properties.

Lehmer generators

- Lehmer obtained the kth number in the sequence by dividing the kth power of an integer a by another integer M and taking the remainder.

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- Random number generators of this form are called Lehmer generators, or multiplicative linear-congruential generators.

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- Independent and uniformly distributed succesive values
  The correlation between successive numbers should be small.

Problems with random number generators

- Research has shown that Lehmer generators obey these properties provided a and M are carefully chosen. However care is needed.
- In the early 1970s most university mainframes were using a linear-congruence generator known as `RANDU`.
- It used the values $a = 65539$ and $M = 2^{31}$.
- Although the output looked random, detailed statistical analysis showed that there was significant correlation in the output.

Efficiency of random number generators

- This form of generator continues to be used, if somewhat more warily.
- These generators are particularly efficient if M is chosen to be a power of 2.
- In this case finding the residue amounts to simply truncating the result of the multiplication.
- However a modulus of the form 2^k results in a shorter cycle: 2^{k-2} at best.

Mersenne Twister

- One of the best families of random number generators for simulation is that based on the Mersenne Twister algorithm.
- It is used by default in python, R, MATLAB and several other languages.
- It comes in a number of variants, but the commonly used MT19937 variant produces a sequence of 32-bit integers, and has the following desirable properties:
  - It has a very long period of $2^{19937} - 1$.
  - It passes numerous tests for statistical randomness, including some stringent tests which are failed by linear congruential random number generators.

Random variate generation algorithms

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A good example is the book by Raj Jain:

The Art of Computer Systems Performance Analysis, Wiley, 1991.

Inverse transformations

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- Inverse transformation algorithms are based on the observation that for any probability distribution with distribution function $F(x)$, the value of $F(x)$ is uniformly distributed between 0 and 1.
- Thus, using values from the random number stream, $u = X_k$, the function is inverted to find the next value of x: $x = F^{-1}(u)$.

Exponential distributions

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Note

Strictly speaking, the equation should be

$$x = -\frac{1}{\lambda} \ln(1 - u)$$

but since u is uniformly distributed between 0 and 1, $1 - u$ will be uniformly distributed between 0 and 1 and the generation algorithm can be simplified.

Boolean-valued distributions

- Boolean-valued distributions which are used to make decisions within a model take a single real parameter, p, such that $0 \leq p \leq 1$.
- This represents the probability of a "positive" outcome.
- Then each time the branching point in the model is reached, the next random number in the stream is generated $u = X_k$.
- If $u \leq p$ the positive branch is taken;
- If $u > p$ the other branch is selected.

Simulation packages

- One of the benefits of using a simulation package is that at least some of these algorithms are provided for us.
- Each time that a distribution is instantiated the seed for the random number generator can be set explicitly.
- If seeds are not well-spaced there may be overlap between the sequences of random numbers used by the generators resulting in correlation between the samples used in the simulation.
- Some simulation packages provide an automatic seeding mechanism which will seed each distribution with a distinct seed which is far in the cycle from other seeds currently in use.

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- It is part of model construction to make sure that all the necessary counters and updates are in place to allow the measures to be collected as the model runs.
- This is sometimes called instrumentation of a model as it is analogous to inserting probes and monitors on a real system.

Simulation trajectories

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So, in general, any estimate for the value of a performance measure generated from a single run constitutes a single observation in the possible sample space.

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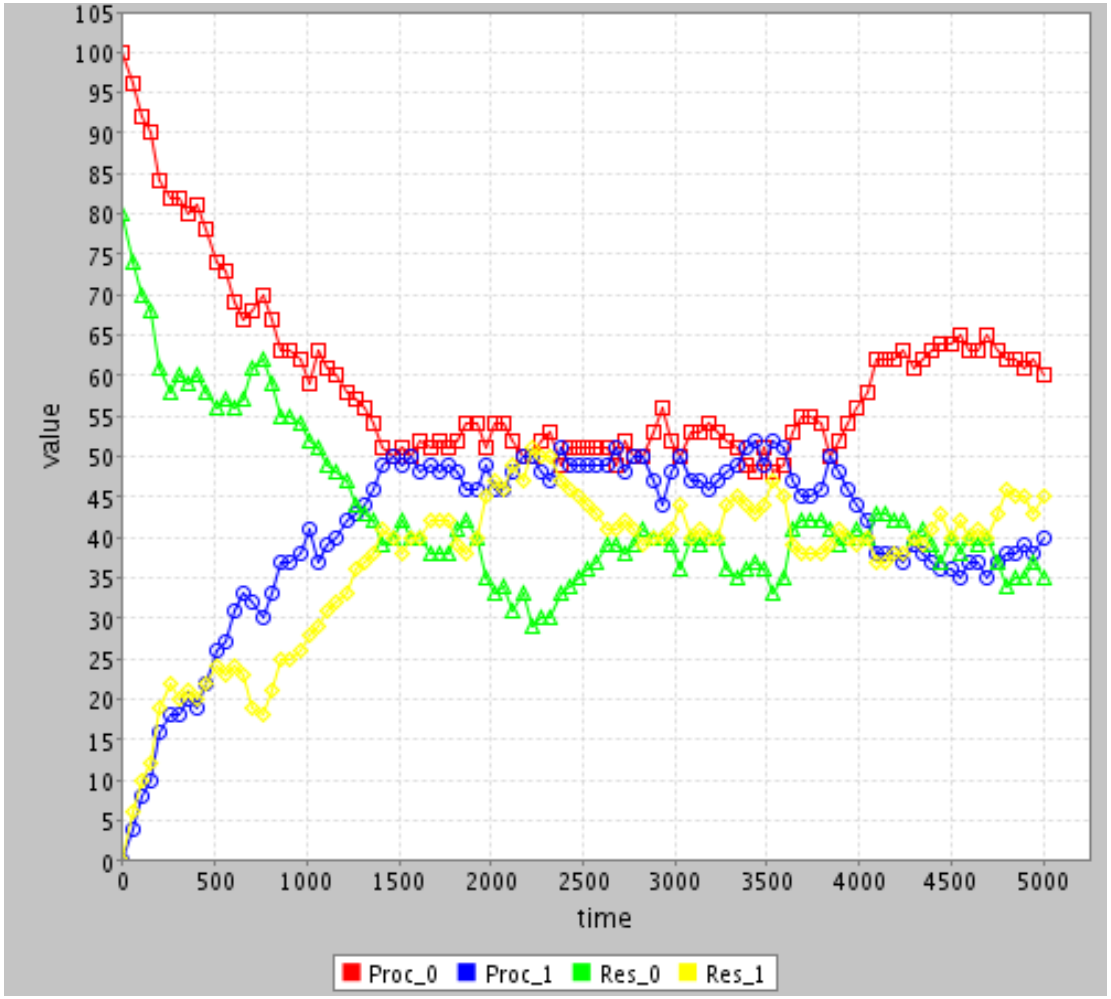
  - choosing the starting state of the simulation;
  - choosing the warm-up period that is allowed to elapse before data collection begins;
  - choosing a run length that ensures that the calculated averages are representative of the unknown true long term average.

Example

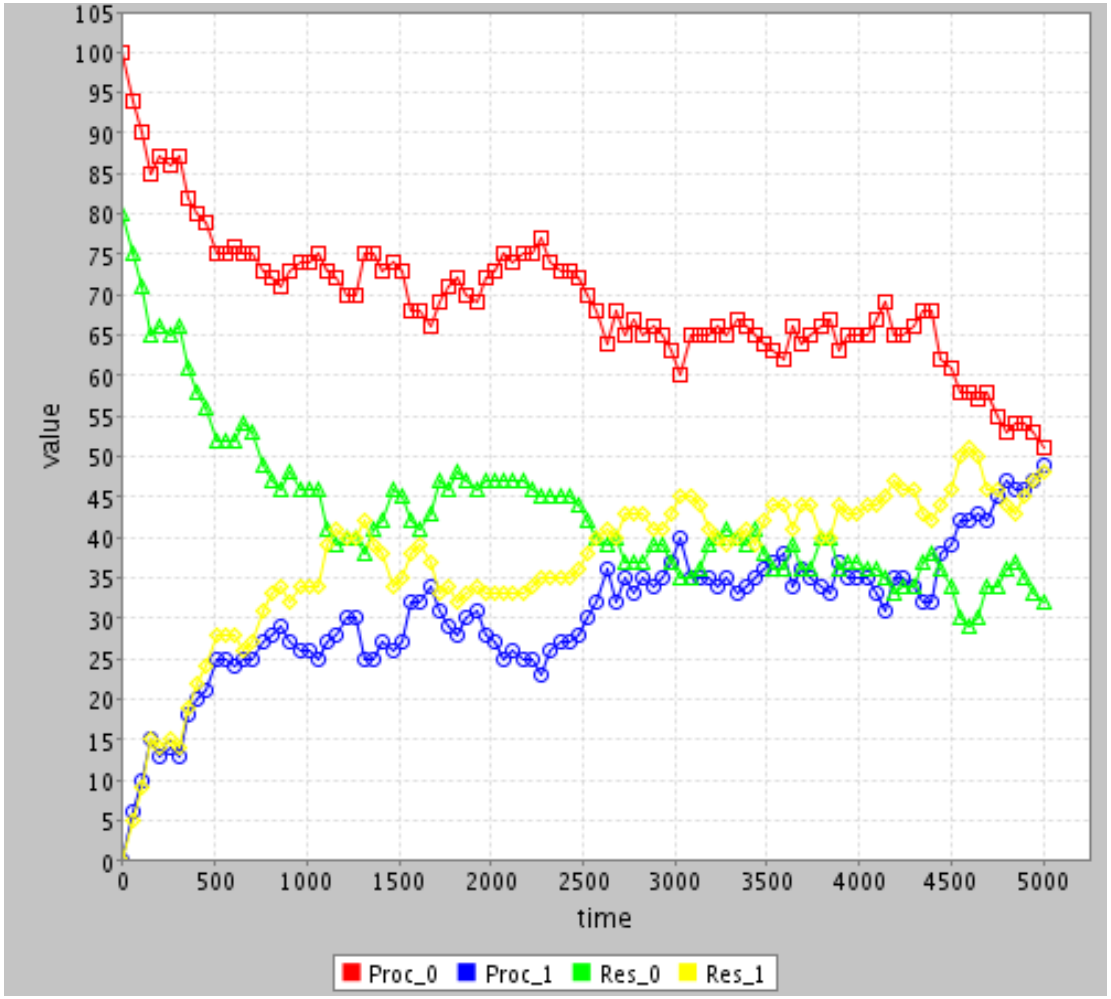
$$
\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
Res_1 &\stackrel{def}{=} (reset, r_4).Res_0
\end{aligned}
$$

$$
Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]
$$

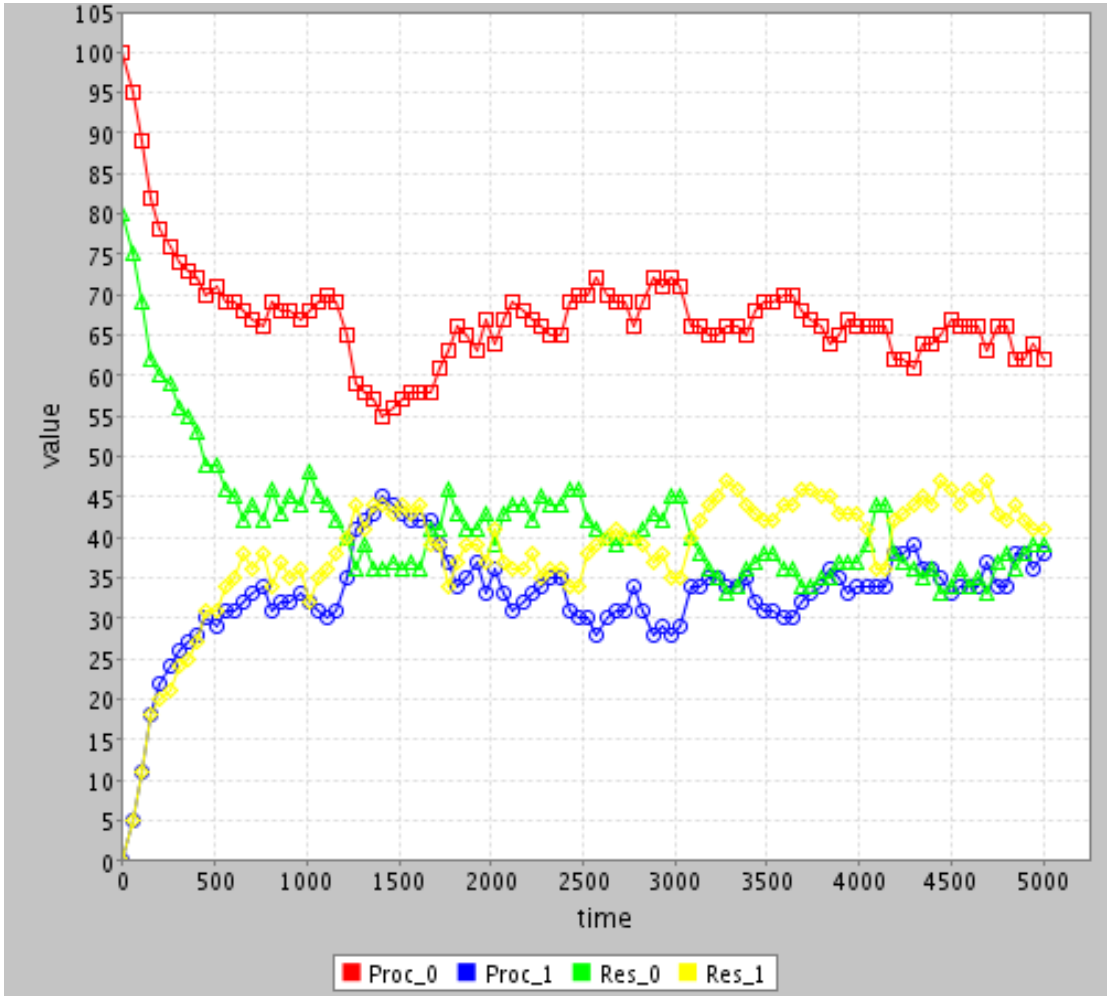
100 processors and 80 resources (simulation run A)



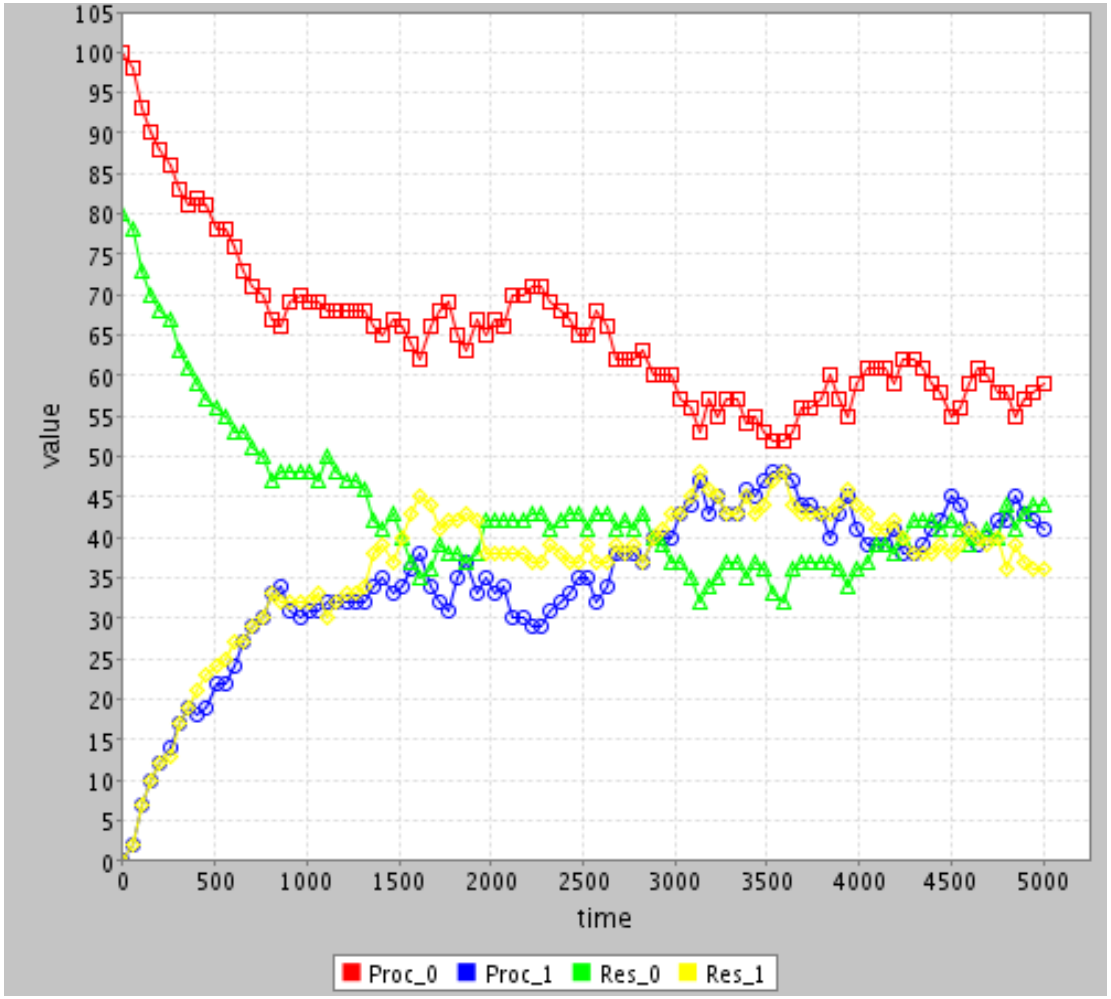
100 processors and 80 resources (simulation run B)



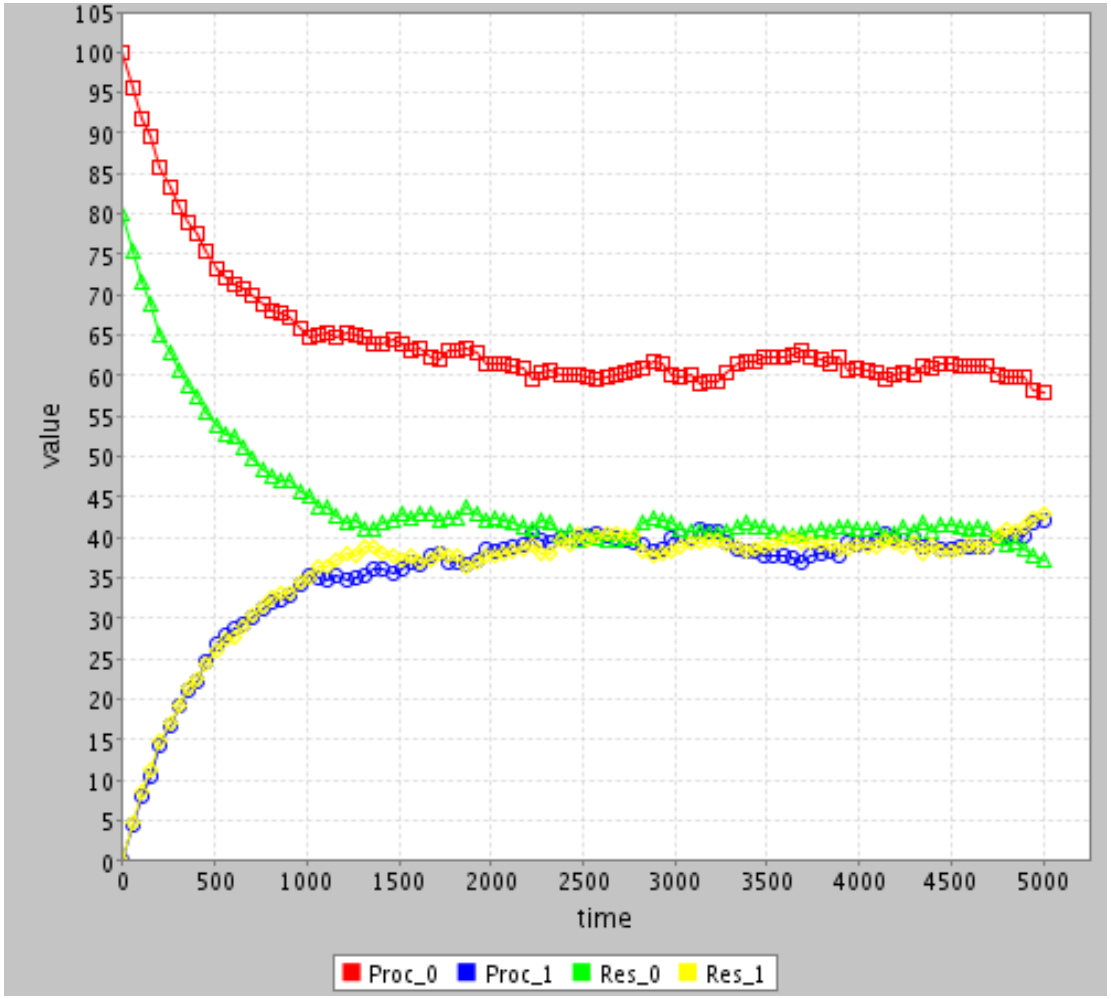
100 processors and 80 resources (simulation run C)



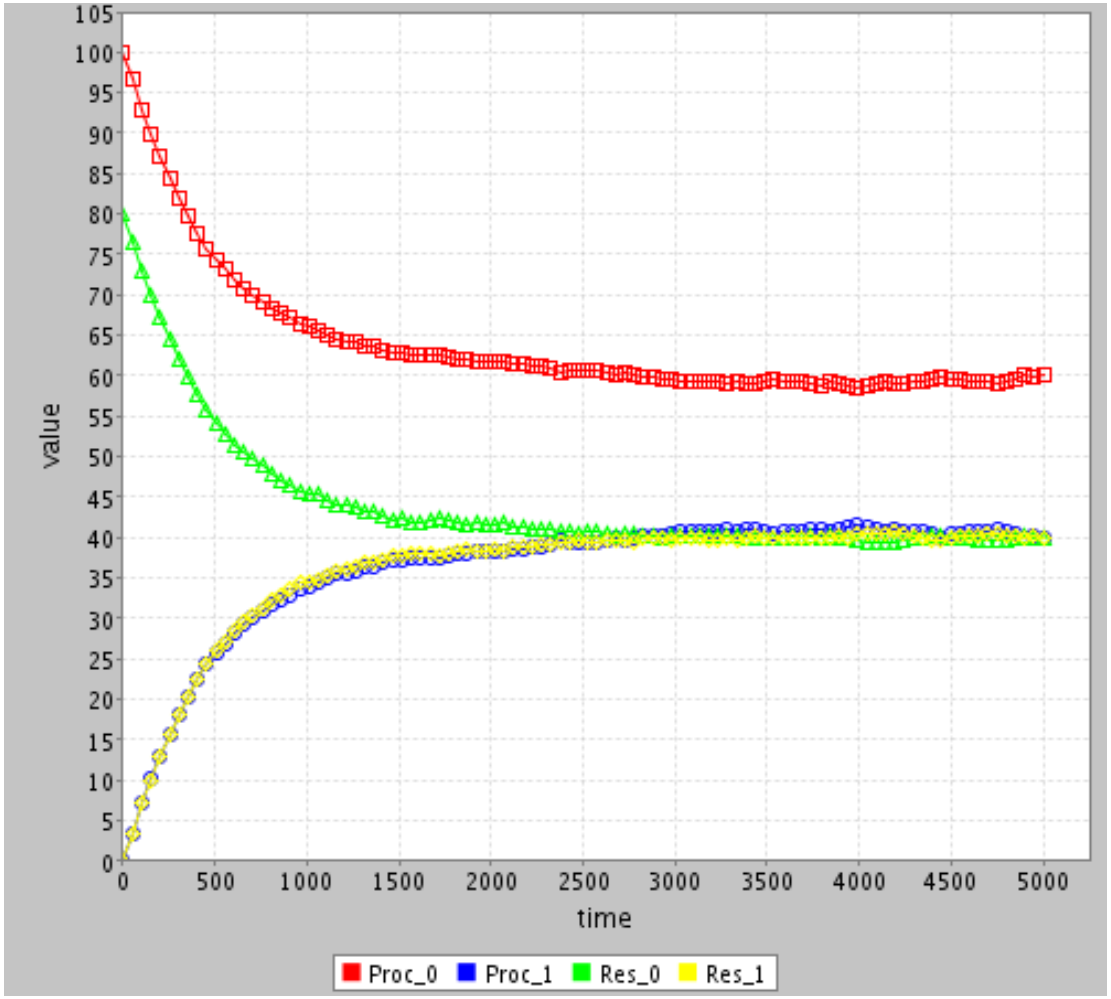
100 processors and 80 resources (simulation run D)



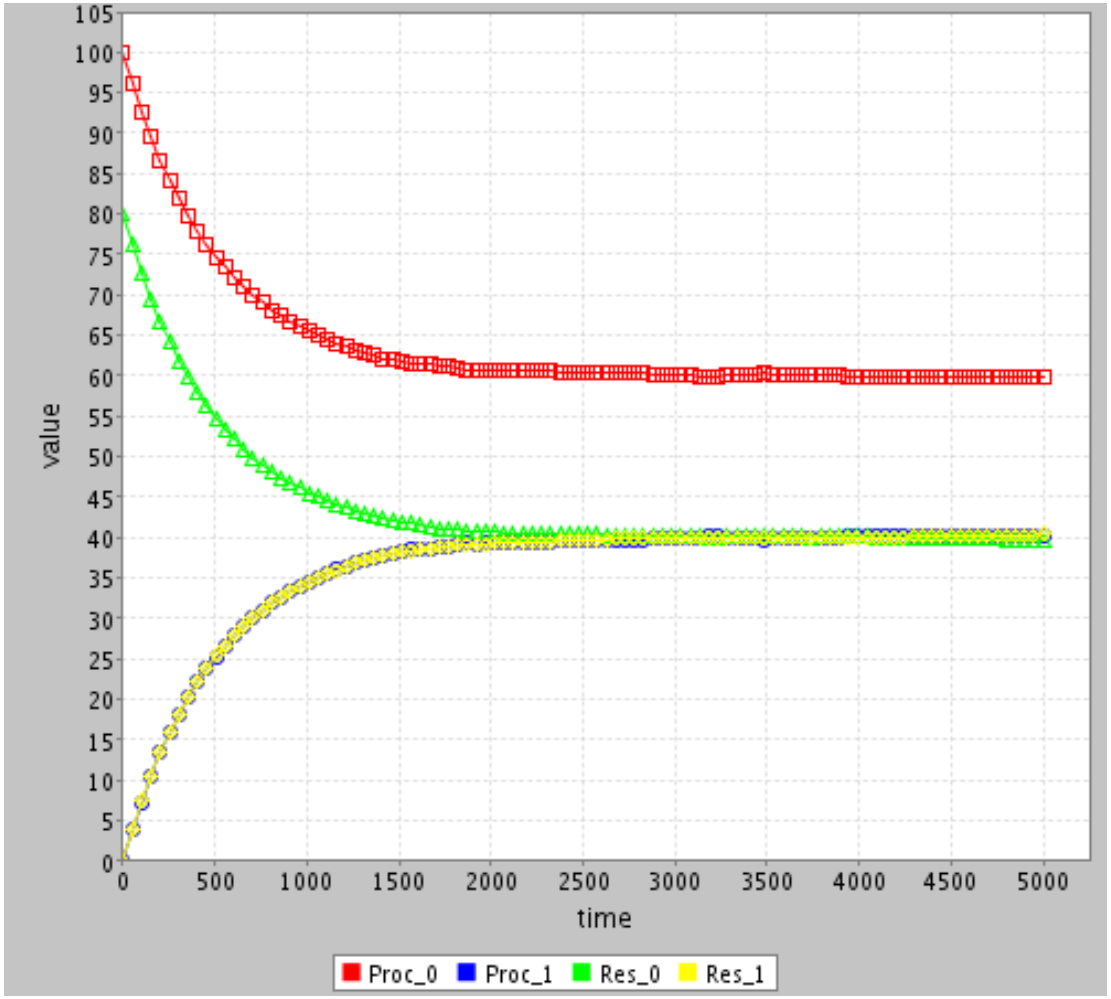
100 processors and 80 resources (average of 10 runs)



100 Processors and 80 resources (average of 100 runs)



100 processors and 80 resources (average of 1000 runs)



Statistical techniques

- Statistical techniques can be used to assess how and when the calculated averages approximate the true average, i.e. to analyse the accuracy of our current estimate.
- This is often done in terms of a confidence interval.
- A confidence interval expresses probabilistic bounds on the error of our current estimate.

Confidence intervals

A confidence interval (c_1, c_2) with confidence level $X\%$, means that with probability $X/100$ the real value v lies between the values c_1 and c_2, i.e.

$$\Pr(c_1 \leq v \leq c_2) = X/100$$

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$$\Pr(c_1 \leq v \leq c_2) = X/100$$

$X/100$ is usually written in the form $1 - \alpha$, and α is called the significance level, and $(1 - \alpha)$ is called the confidence coefficient.

Confidence intervals and variance

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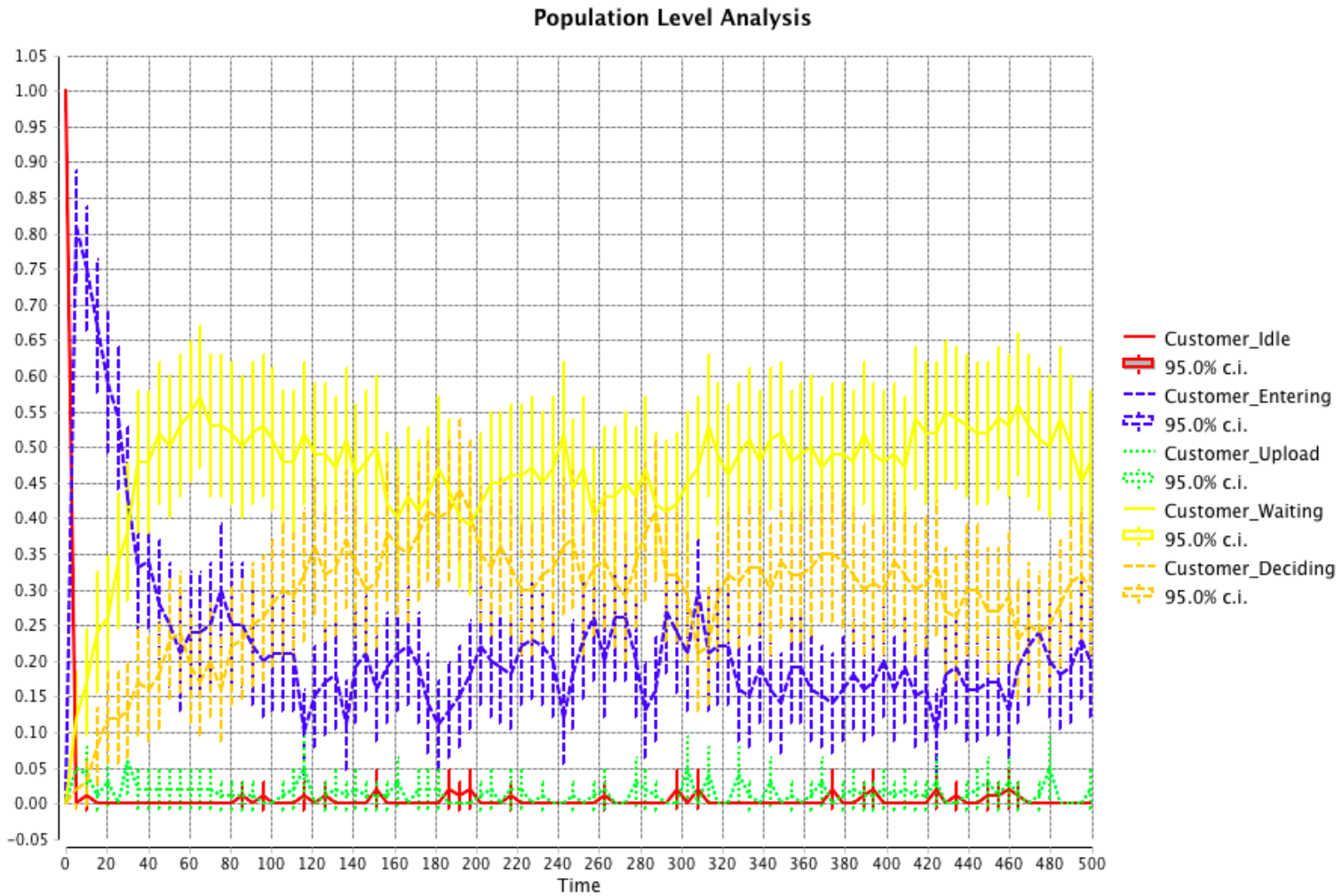
The greater the variance, the wider the confidence interval; the smaller the variance, the tighter the bounds.

Confidence interval example



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For example, if we wish to investigate how many messages can be processed by a dealers' transaction processing system in the first hour of trading then it makes sense to run the model for 3600 seconds.

However, if the question is how many messages can be processed in an average hour then running the model for 3600 seconds is unlikely to be enough.

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For this type of simulation the question becomes how many times the simulation must be repeated (with different random number streams) to achieve a required confidence interval.

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In other words we are theoretically interested in the observations obtained from runs of the model which are infinitely long.

However, in practice we are interested in finite run lengths and estimating the steady state distribution of the measures we are interested in from finitely many samples.

Initial conditions, bias

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Thus the modeller must make some effort to remove the effect of the starting state, sometimes termed bias, from the sample data used for estimating the performance measure of interest.

Unfortunately there is no precise procedure for this as we cannot generally detect when the model has moved from transient behaviour (the warm-up period) to steady state behaviour.

Heuristics for reducing bias

The common techniques are

1. Long runs.
2. Proper initialisation.
3. Truncation.
4. Initial data deletion.
5. Moving average of independent replications.
6. Batch means.

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The last four techniques are all based on the assumption that variability is less during steady state behaviour than during transient behaviour.

Options for terminating a simulation

Option 1

- begin the simulation at time 0
- begin data collection at specified time $w \geq 0$
- complete data collection at specified time $w + t$
- terminate execution of the simulation at time $w + t$
- calculate summary statistics based on sample path data collected in the time interval $(w, w + t)$.

Options for terminating a simulation

Option 2

- begin the simulation at time 0
- begin data collection when the Mth event completes
- complete data collection when the $(M+N)$th event completes
- terminate execution of the simulation when the $(M+N)$th event completes
- calculate summary statistics based on sample path data collected in the time interval (t_M, t_{M+N}), where t_j is the time at which the jth event completes.

Advantages and disadvantages

- Option 1 implies that the simulated time $(w, w + t)$ for data collection is predetermined but the number of event completions is random.
- Conversely, Option 2 implies that the time period for data collection is random but the number of event completions is predetermined.
- In queueing networks, Option 1 is preferable for calculating queue lengths and resource utilisations, whereas Option 2 is preferable for calculating waiting times.

Variance reduction techniques

- Assume that we are running a simulation model in order to estimate some performance measure M.
- During the ith execution of the model we make observations of M, o_{ij} and at the end of the run we calculate the mean value of the observations O_i.
- Note that the observations o_{ij} in most simulations are not independent. Successive observations are often correlated.

Example of correlation

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Note

This is why, in general, a simulation model must be run several times.

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- Let O denote the mean value of the retained observations, O_i, after m runs.
- The variance over all observations is calculated as:

$$V = \frac{1}{m-1} \sum_{i=1}^{m} (O_i - O)^2$$

Independent replications and steady-state

For steady-state analysis independent replication is an inefficient way to generate samples, since for each sample point, O_i, k observations, $\{o_{i1}, \ldots, o_{ik}\}$, must be discarded.

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- Each sample point O_i is the mean generated from a subset of observations S_i, and O is the mean generated from the O_i.

Batch means

- In the method of batch means the model is run only once but for a very long period.
- The run is divided into a series of sub-periods of length ℓ, and measures over each sub-run form a single point estimate.
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- Each sample point O_i is the mean generated from a subset of observations S_i, and O is the mean generated from the O_i.
- Variance is calculated as above.

Batch means and independence

This method is unreliable since the sub-periods are clearly not independent.

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However it has the advantage that only one set of observations $\{o_i \ldots o_k\}$ needs to be discarded to overcome the warm-up effects in steady state analysis.

Regeneration

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- The behaviour of the model (queue length, waiting time etc) after a visit to such a state does not depend on the previous history of the model in any way.
- The duration between two successive regeneration points is called a regeneration cycle.

Variance computation and regeneration

- The variance computation using regeneration cycles is a bit more complex than that in the method of batch means or the method of independent replications.
- This is because the regeneration cycles are of different lengths, whereas in the other two methods the batches or replications are all of the same length.

Regeneration considered

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Another disadvantage is that it is not possible to define the length of a simulation run beforehand.