Performance Modelling — Lecture 11
PEPA Case Study

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- In this graph, each node is a state of the model (comprised of the local states of each of the components) and the arcs represent the actions causing the move from one state to another.
- This graph can be treated as the state transition diagram of a CTMC, leading to the generation of the infinitesimal generator matrix.

Upgrading a PC LAN

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A node can transmit, only whilst it holds the token.

Upgrading a PC LAN

There are currently four PCs (or similar devices) connected to the LAN in a small office, but the company has recently recruited two new employees, each of whom will have a PC. Our task is to find out how the delay experienced by data packets at each PC will be affected if another two PCs are added.

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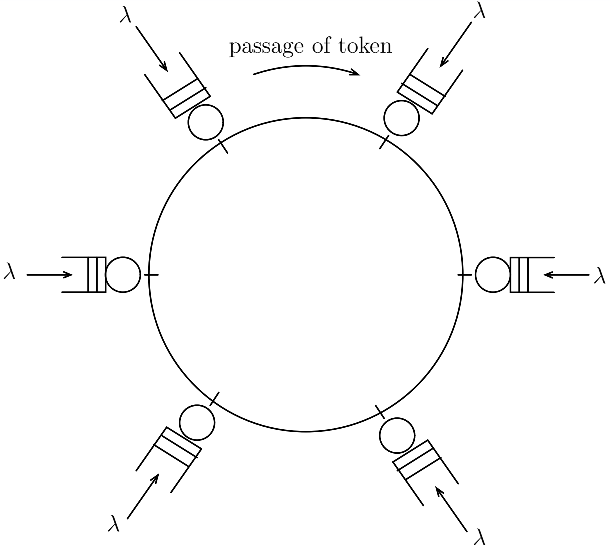
- Transmission is gated: each PC can transmit at most one data packet per visit of the token.


λ
λ
passage of token
λ
λ
λ
λ

Modelling the system: choosing components

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We will need another component to represent the medium. As remarked previously, the medium can be represented solely by the token.

Modelling the system: choosing actvities

The description of the PC is very simple in this case. It only has two activities which it can undertake:

- generate a data packet;
- transmit a data packet.

Moreover we are told that it can only hold one data packet at a time and so these activities must be undertaken sequentially.

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This suggests the following PEPA component for the ith PC:

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This will need some refinement when we consider interaction with the token.

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- transmit a token if there is one to transmit and then walk on; or
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$$\mathit{Token}_i \stackrel{\mathit{def}}{=} (\mathit{walkon}_{i+1}, \omega).\mathit{Token}_{i+1} + (\mathit{transmit}_i, \mu).(\mathit{walk}_{i+1}, \omega).\mathit{Token}_{i+1}$$

Refining the components

In order to ensure that the token's choice is made dependent on the state of PC being visited, we add a walkon action to the PC when it is empty, and impose a cooperation between the PC and the Token for both walkon and serve.

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Complete model: four PC case

$$PC_{10} \stackrel{def}{=} (arrive, \lambda).PC_{11} + (walkon_2, \omega).PC_{10}$$
$$PC_{11} \stackrel{def}{=} (transmit_1, \mu).PC_{10}$$

$$PC_{20} \stackrel{def}{=} (arrive, \lambda).PC_{21} + (walkon_3, \omega).PC_{20}$$
$$PC_{21} \stackrel{def}{=} (transmit_2, \mu).PC_{20}$$

$$PC_{30} \stackrel{def}{=} (arrive, \lambda).PC_{31} + (walkon_4, \omega).PC_{30}$$
$$PC_{31} \stackrel{def}{=} (transmit_3, \mu).PC_{30}$$

$$PC_{40} \stackrel{def}{=} (arrive, \lambda).PC_{41} + (walkon_1, \omega).PC_{40}$$
$$PC_{41} \stackrel{def}{=} (transmit_4, \mu).PC_{40}$$

$$Token_1 \stackrel{def}{=} (walkon_2, \omega).Token_2 + (transmit_1, \mu).(walk_2, \omega).Token_2$$
$$Token_2 \stackrel{def}{=} (walkon_3, \omega).Token_3 + (transmit_2, \mu).(walk_3, \omega).Token_3$$
$$Token_3 \stackrel{def}{=} (walkon_4, \omega).Token_4 + (transmit_3, \mu).(walk_4, \omega).Token_4$$
$$Token_4 \stackrel{def}{=} (walkon_1, \omega).Token_1 + (transmit_4, \mu).(walk_1, \omega).Token_1$$

$$LAN \stackrel{def}{=} (PC_{10} \parallel PC_{20} \parallel PC_{30} \parallel PC_{40}) \bowtie_{L} Token_1$$
$$\text{where } L = \{walkon_1, walkon_2, walkon_3, walkon_4, serve_1, serve_2, serve_3, serve_4\}.$$

Here we have arbitrarily chosen a starting state in which all the PCs are empty and the Token is at PC1.

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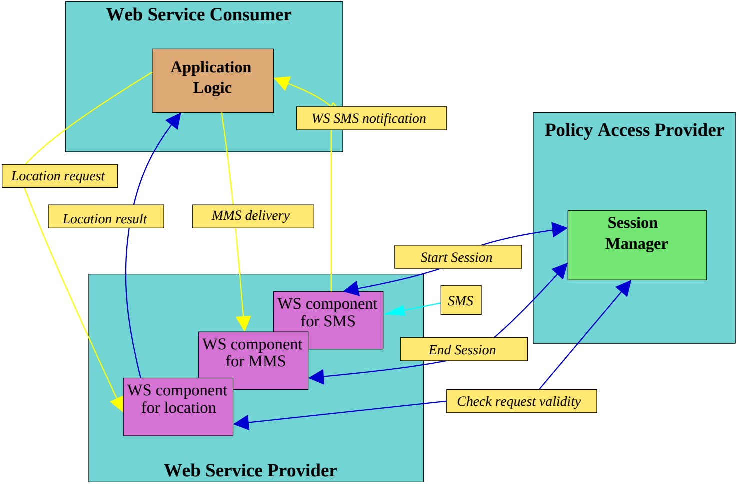
Moreover the service provider imposes a restriction that only one request may be handled for each SMS message received.

Schematic view



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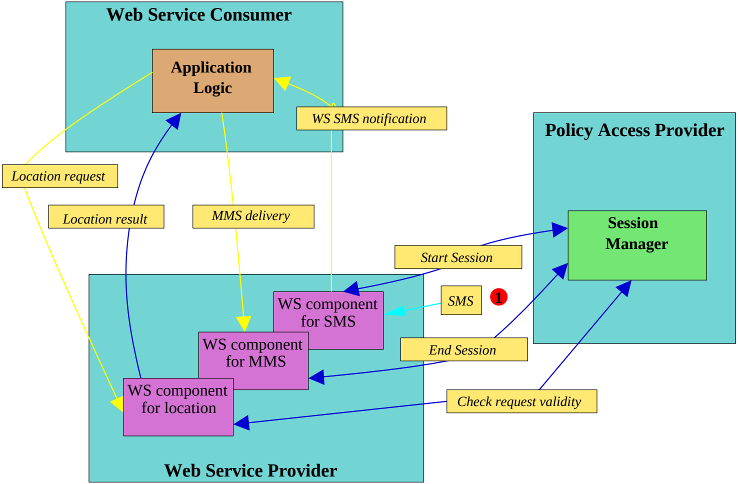

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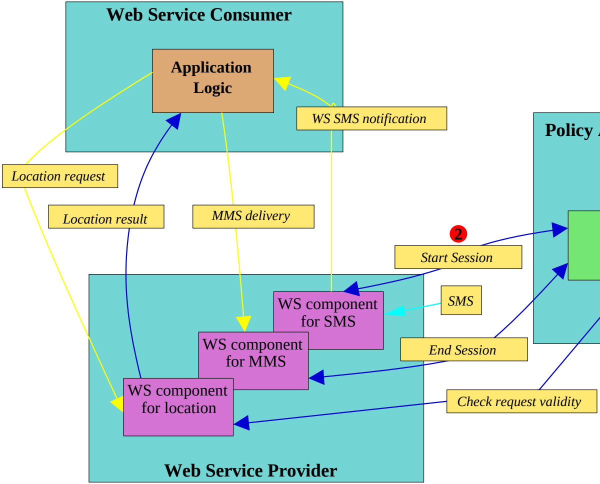

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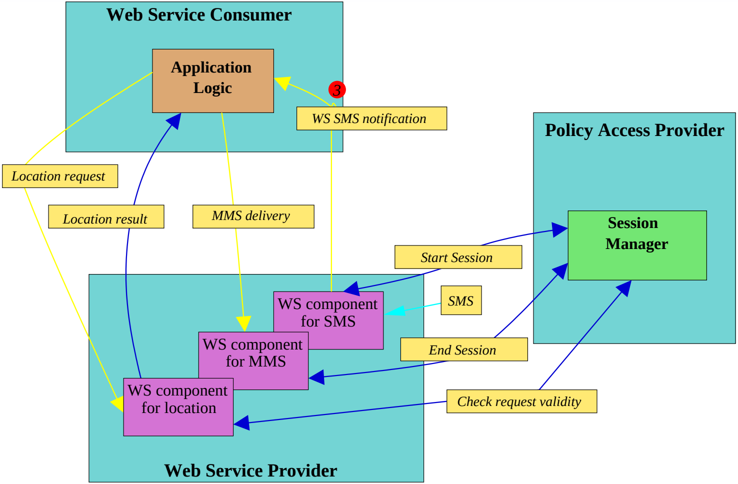

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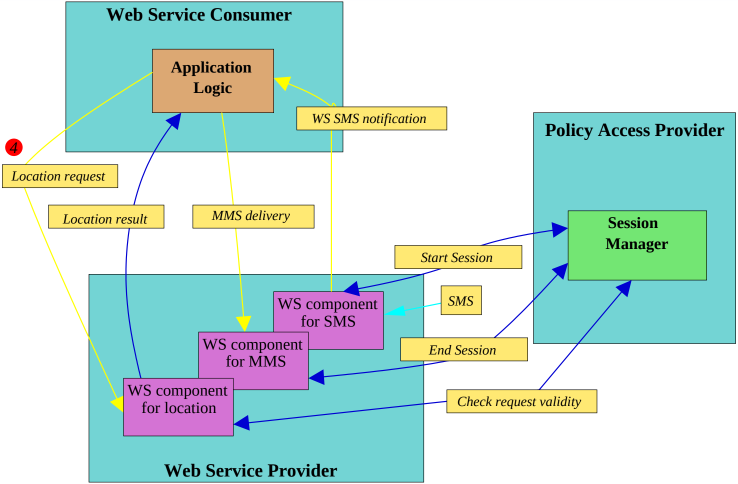

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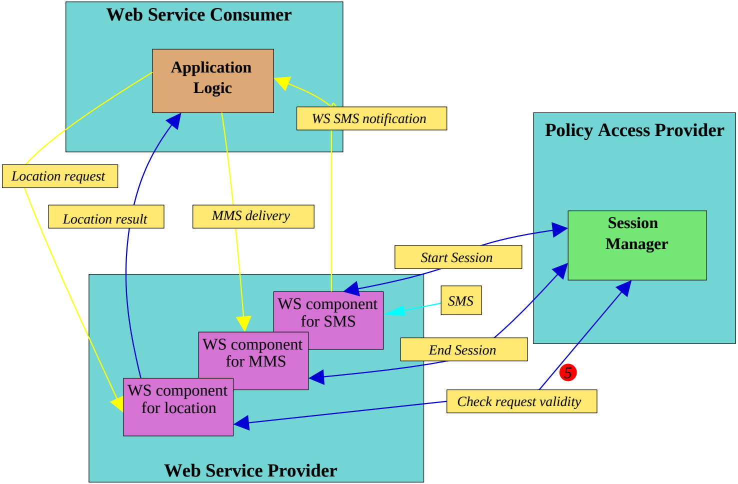

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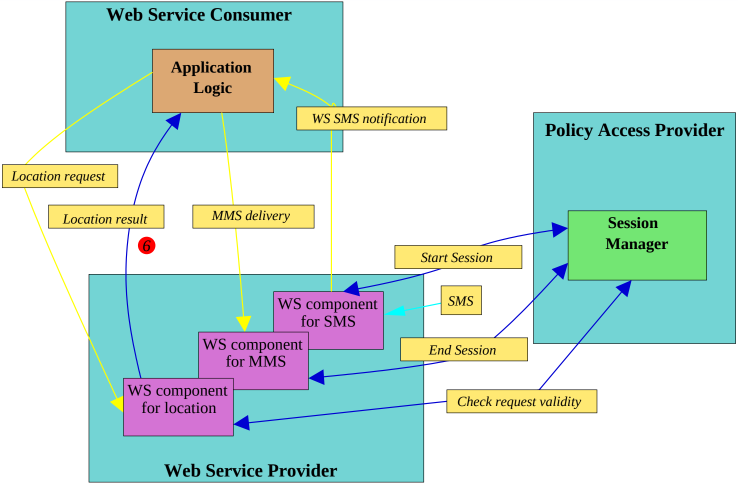

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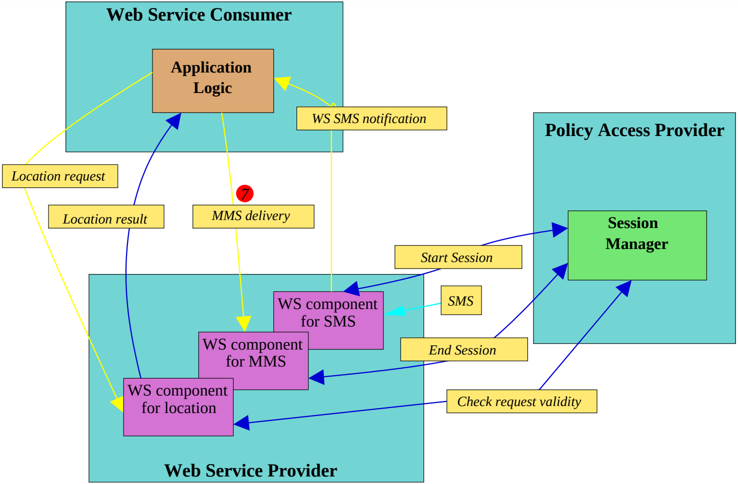

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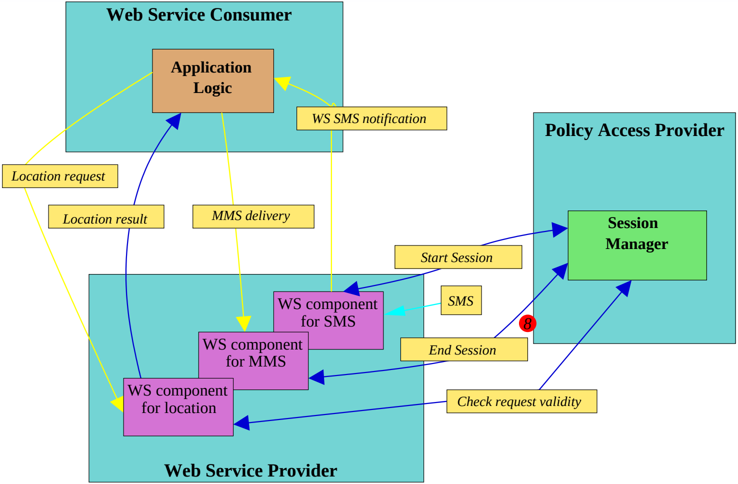

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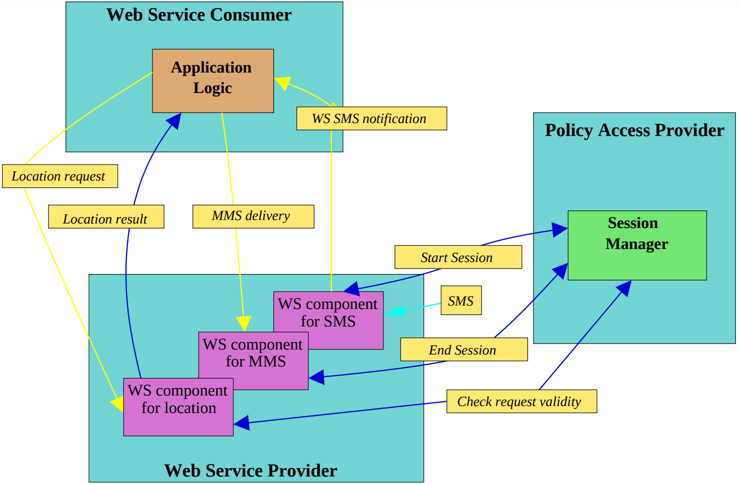

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Concurrency is introduced into the model by allowing multiple sessions rather than by representing the constituent web services separately.

Component *Customer*

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We associate the user-perceived system performance with the throughput of the *getMap* action which can be calculated directly from the steady state probability distribution of the underlying Markov chain.

Component *WSConsumer*

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$$
\begin{aligned}
WSConsumer &\stackrel{def}{=} (notify, \top).WSConsumer_2 \\
WSConsumer_2 &\stackrel{def}{=} (locReq, r_4).WSConsumer_3 \\
WSConsumer_3 &\stackrel{def}{=} (locRes, \top).WSConsumer_4 \\
&\;\;+\;\; (locErr, \top).WSConsumer \\
WSConsumer_4 &\stackrel{def}{=} (compute, r_7).WSConsumer_5 \\
WSConsumer_5 &\stackrel{def}{=} (sendMMS, r_9).WSConsumer
\end{aligned}
$$

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- If the check is successful the location must be returned to the Web Service Consumer in the form of a map (*getMap*).
- If the check revealed an invalid request (*locErr*) then an error must be returned to the Web Service Consumer (*get404*) and the session terminated (*stopSession*).

Component *WSProvider*

$$
\begin{aligned}
WSProvider &\stackrel{def}{=} (getSMS, \top).WSProvider_2 \\
WSProvider_2 &\stackrel{def}{=} (startSession, r_2).WSProvider_3 \\
WSProvider_3 &\stackrel{def}{=} (notify, r_3).WSProvider_4 \\
WSProvider_4 &\stackrel{def}{=} (locReq, \top).WSProvider_5 \\
WSProvider_5 &\stackrel{def}{=} (checkValid, 99 \cdot \top).WSProvider_6 \\
&+ (checkValid, \top).WSProvider_{10}
\end{aligned}
$$

Component *WSProvider* cont.

$$
\begin{aligned}
WSProvider_6 &\stackrel{def}{=} (locRes, r_6).WSProvider_7 \\
WSProvider_7 &\stackrel{def}{=} (sendMMS, \top).WSProvider_8 \\
WSProvider_8 &\stackrel{def}{=} (getMap, r_8).WSProvider_9 \\
WSProvider_9 &\stackrel{def}{=} (stopSession, r_2).WSProvider \\
WSProvider_{10} &\stackrel{def}{=} (locErr, r_6).WSProvider_{11} \\
WSProvider_{11} &\stackrel{def}{=} (get404, r_8).WSProvider_9
\end{aligned}
$$

Component *PAProvider*

We consider a stateless implementation of the policy access provider.

$$
\begin{aligned}
\mathit{PAProvider} &\stackrel{\mathit{def}}{=} (\mathit{startSession}, \top).\mathit{PAProvider} \\
&+ (\mathit{checkValid}, r_5).\mathit{PAProvider} \\
&+ (\mathit{stopSession}, \top).\mathit{PAProvider}
\end{aligned}
$$

Model Component *WSComp*

The complete system is composed of some number of instances of the components interacting on their shared activities:

$$
\begin{aligned}
WSComp \stackrel{def}{=} & ((Customer[N_C] \underset{L_1}{\bowtie} WSProvider[N_{WSP}]) \\
& \underset{L_2}{\bowtie} WSConsumer[N_{WSC}]) \\
& \underset{L_3}{\bowtie} PAProvider[N_{PAP}]
\end{aligned}
$$

where the cooperation sets are

$$
\begin{aligned}
L_1 &= \{getSMS, getMap, get404\} \\
L_2 &= \{notify, locReq, locRes, locErr, sendMMS\} \\
L_3 &= \{startSession, checkValid, stopSession\}
\end{aligned}
$$

Parameter Values

param.	*value*	*explanation*
r_1	0.0010	rate customers request maps
r_2	0.5	rate session can be started
r_3	0.1	notification exchange between consumer and provider
r_4	0.1	rate requests for location can be satisfied
r_5	0.05	rate the provider can check the validity of the request
r_6	0.1	rate location information can be returned to consumer
r_7	0.05	rate maps can be generated
r_8	0.02	rate MMS messages can be sent from provider to customer
r_9	$10.0 * r_8$	rate MMS messages can be sent via the Web Service

Steady State Analysis for System Tuning

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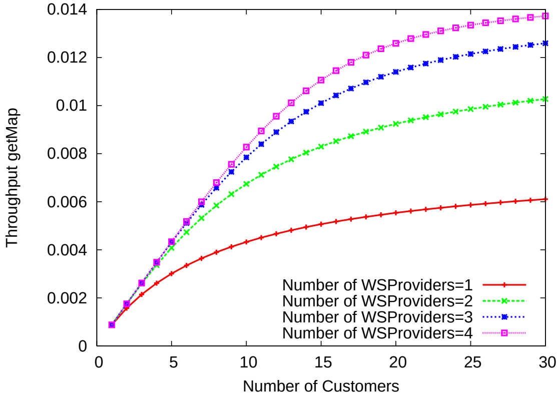
- Suppose that we want to design the system in such a way that it can handle 30 independent customers.
- Some parameters such as the network delays may be constrained by the available technology.
- However, there are a number of degrees of freedom which let us vary, for example, the number of threads of control of the components of the system.
- The aim of the analysis is to deliver a satisfactory service in a cost-effective way.
- The simplest example of a cost function may be a linear dependency on the number of copies of a component or the rate at which an activity is performed.

Throughput of the *getMap* action

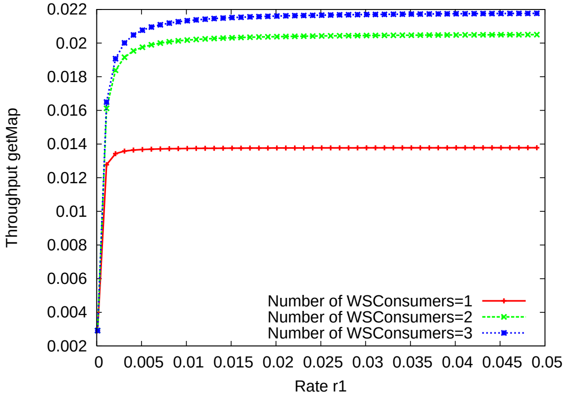


as the number of customers varies between 1 and 30 for various numbers of copies of the *WSProvider* component.

Throughput of the *getMap* action

- Under heavy load increasing the number of providers initially leads to a sharp increase in the throughput. However the gain deteriorates so that the system with four copies is just 8.7% faster than the system with three.
- In the following we settle on three copies of *WSProvider*.

Throughput of *getMap* action



as the request arrival rate (r_1) varies for differing numbers of *WSConsumer*.

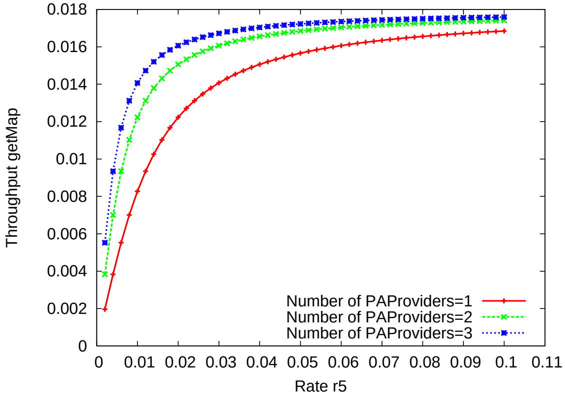
Throughput of *getMap* action

- Every line starts to plateau at approximately $r_1 = 0.010$ following an initial sharp increase. This suggests that the user is the bottleneck in the system when the arrival rate is lower. Conversely, at high rates the system becomes congested.
- Whilst having two copies of *WSConsumer*, corresponding to two operating threads of control, improves performance significantly, the subsequent increase with three copies is less pronounced.
- So we set the number of copies of *WSConsumer* to 2.

Optimising the number of copies of *PAProvider*

- Here we are particularly interested in the overall impact of the rate at which the validity check is performed.
- Slower rates may mean more computationally expensive validation.
- Faster rates may involve less accuracy and lower security of the system.

Throughput of *getMap* action



as the validity check rate (r_5) varies for differing numbers of *PAProvider*.

Throughput of *getMap* action

- A sharp increase followed by a constant levelling off suggests that optimal rate values lie on the left of the plateau, as faster rates do not improve the system considerably.
- As for the optimal number of copies of *PAProvider*, deploying two copies rather than one dramatically increases the quality of service of the overall system.
- With a similar approach as previously discussed, the modeller may want to consider the trade-off between the cost of adding a third copy and the throughput increase.

An alternative design for *PAProvider*

- The original design of *PAProvider* is stateless.
- Any of its services can be called at any point, the correctness of the system being guaranteed by implementation-specific constraints such as session identifiers being uniquely assigned to the clients and passed as parameters of the method calls.

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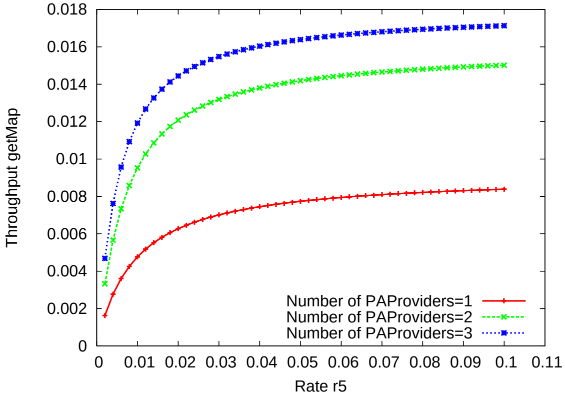
- Any of its services can be called at any point, the correctness of the system being guaranteed by implementation-specific constraints such as session identifiers being uniquely assigned to the clients and passed as parameters of the method calls.
- Alternatively we may consider a stateful implementation, modelled as a sequential component with three local states.
- This implementation has the consequence that there can never be more than N_{PAP} *WSProvider* which have started a session with a *PAProvider*

Component *PAProvider* — Stateful Version

It maintains a thread for each session and carries out the validity check on behalf of the Web Service Provider.

$$
\begin{aligned}
PAProvider &\stackrel{def}{=} (startSession, \top).PAProvider_2 \\
PAProvider_2 &\stackrel{def}{=} (checkValid, r_5).PAProvider_3 \\
PAProvider_3 &\stackrel{def}{=} (stopSession, \top).PAProvider
\end{aligned}
$$

Throughput of *getMap* action



as the validity check rate (r_5) varies for differing numbers of *PAProvider* (stateful version).

Throughput of *getMap* action

- In this case the incremental gain in adding more copies has become more marked.
- However, the modeller may want to prefer the original version, as three copies of the stateful provider deliver about as much as the throughput of only one copy of the stateless implementation.

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