Performance Modelling — Lecture 12(ish)
PEPA Exercises

Jane Hillston
School of Informatics
The University of Edinburgh
Scotland

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A basic PEPA model

Consider the web service previous modelled with GSPN:

A third party app receives requests from users for live bus positioning information. It sends requests to the the Google Map API and the TfE Bus Info API and then aggregates the results to present a map view of the bus data which is returned to the user.

Construct a PEPA model to represent this system.

PEPA model

$$
\begin{aligned}
User &\stackrel{def}{=} (bus\_pos\_req, r).(bus\_pos\_resp, \top).User \\
Map\_finder &\stackrel{def}{=} (bus\_pos\_req, r).(google\_req, \lambda_1). \\
&\qquad (google\_resp, \top).(bus\_pos\_resp, \top).Map\_finder \\
Bus\_finder &\stackrel{def}{=} (bus\_pos\_req, r).(tfe\_req, \lambda_2). \\
&\qquad (tfe\_resp, \top).(bus\_pos\_resp, \top).Bus\_finder \\
Google &\stackrel{def}{=} (google\_req, \top).(google\_resp, \mu_1).Google \\
TfE &\stackrel{def}{=} (tfe\_req, \top).(tfe\_resp, \mu_2).Tfe \\
System &\stackrel{def}{=} User \underset{L}{\bowtie} \Big(Bus\_finder \underset{L}{\bowtie} Map\_finder) \underset{K}{\bowtie} (Google \parallel TfE)\Big)
\end{aligned}
$$

where $L = \{bus\_pos\_req, bus\_pos\_resp\}$ and
$K = \{google\_req, google\_resp, (tfe\_req, \top).(tfe\_resp, \mu_2)\}$.

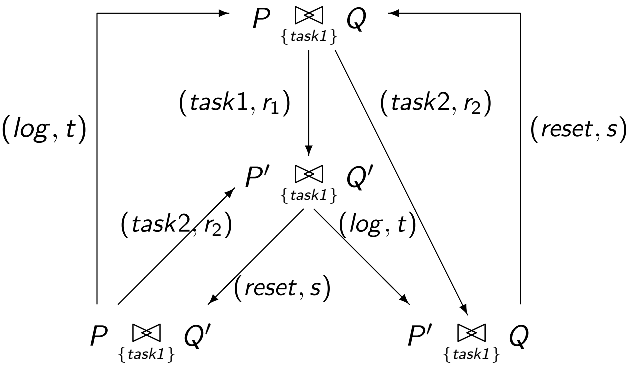
Derivative graph

Consider the following PEPA model:

$$
\begin{aligned}
P &\stackrel{\mathit{def}}{=} (\mathit{task1}, r_1).P' + (\mathit{task2}, r_2).P' \\
P' &\stackrel{\mathit{def}}{=} (\mathit{reset}, s).P \\
Q &\stackrel{\mathit{def}}{=} (\mathit{task1}, \top).Q' \\
Q' &\stackrel{\mathit{def}}{=} (\mathit{log}, t).Q \\
\mathit{Sys} &\stackrel{\mathit{def}}{=} P \underset{\{\mathit{task1}\}}{\bowtie} Q
\end{aligned}
$$

1. Construct the derivative graph or labelled transition system corresponding to this model.
2. Construct the infinitesimal generator matrix for the CTMC underlying this model.

Solution: derivation graph



Solution: Infinitesimal generator matrix

Associating derivatives to states as follows:

$$
\begin{array}{lllllll}
X_0 & \leftrightarrow & P \underset{\{\mathit{task1}\}}{\bowtie} Q & \quad & X_1 & \leftrightarrow & P' \underset{\{\mathit{task1}\}}{\bowtie} Q' \\
X_2 & \leftrightarrow & P \underset{\{\mathit{task1}\}}{\bowtie} Q' & \quad & X_3 & \leftrightarrow & P' \underset{\{\mathit{task1}\}}{\bowtie} Q
\end{array}
$$

we get the infinitesimal generator matrix

$$
Q = \begin{pmatrix}
-(r_1 + r_2) & r_1 & 0 & r_2 \\
0 & -(s + t) & s & t \\
t & r_2 & -(r_2 + t) & 0 \\
s & 0 & 0 & -s
\end{pmatrix}
$$

Multiway synchronisation

The cooperation sets can make a big difference in the behaviour.

Consider the three PEPA models below and explain who will participate in α activities when they occur:

- $\Big((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q\Big) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$
- $((\alpha, r).P \parallel (\alpha, s).Q) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$
- $\Big((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q\Big) \parallel (\alpha, t).R$

Solution

- $\left((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q\right) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$ Here the only α activity will be a cooperation between all three components, which will be at rate $min(r, s, t)$.

Solution

- $\left((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q\right) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$ Here the only α activity will be a cooperation between all three components, which will be at rate $min(r, s, t)$.
- $((\alpha, r).P \parallel (\alpha, s).Q) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$ Here there are two possible α cooperation activities: the cooperation at rate $min(r, t)$, resulting in $(P \parallel (\alpha, s).Q) \underset{\{\alpha\}}{\bowtie} R$. or the cooperation at rate $min(s, t)$, resulting in $((\alpha, r).P \parallel Q) \underset{\{\alpha\}}{\bowtie} R$.

Solution

- $\left((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q\right) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$ Here the only α activity will be a cooperation between all three components, which will be at rate $min(r, s, t)$.

- $((\alpha, r).P \parallel (\alpha, s).Q) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$ Here there are two possible α cooperation activities: the cooperation at rate $min(r, t)$, resulting in $(P \parallel (\alpha, s).Q) \underset{\{\alpha\}}{\bowtie} R$. or the cooperation at rate $min(s, t)$, resulting in $((\alpha, r).P \parallel Q) \underset{\{\alpha\}}{\bowtie} R$.

- $\left((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q\right) \parallel (\alpha, t).R$ Again there are two possible α activities. One is the cooperation between the leftmost components, with rate $min(r, s)$, resulting in $\left(P \underset{\{\alpha\}}{\bowtie} Q\right) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$ and an individual activity, with rate t, resulting in $\left((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q\right) \underset{\{\alpha\}}{\bowtie} R$

Rates of cooperation

Consider the PEPA process below:

$$((\alpha, 4r).A \underset{\{\alpha\}}{\bowtie} (\alpha, r).B) \underset{\{\alpha\}}{\bowtie} ((\alpha, r).C + (\alpha, r).D)$$

What is the apparent rate of α in the process shown above? Explain your reasoning.

Solution

$$((\alpha, 4r).A \underset{\{\alpha\}}{\bowtie} (\alpha, r).B) \underset{\{\alpha\}}{\bowtie} \big((\alpha, r).C + (\alpha, r).D\big)$$

The α activity is a cooperation involving all three components.

- $((\alpha, 4r).A \underset{\{\alpha\}}{\bowtie} (\alpha, r).B)$:
  the rate of α in this component is $min(4r, r) = r$.
- $\big((\alpha, r).C + (\alpha, r).D\big)$:
  the rate of α in the choice is the sum of the rates,
  i.e. $r + r = 2r$
- $((\alpha, 4r).A \underset{\{\alpha\}}{\bowtie} (\alpha, r).B) \underset{\{\alpha\}}{\bowtie} \big((\alpha, r).C + (\alpha, r).D\big)$:
  the rate of the α for the cooperation is the $min(r, 2r) = r$

So the rate of the α in this component is r.