# *The Situation Calculus and the Frame Problem*

Using Theorem Proving to Generate Plans

## Literature

- Malik Ghallab, Dana Nau, and Paolo Traverso. *Automated Planning – Theory and Practice*, section 12.2. Elsevier/Morgan Kaufmann, 2004.
- Murray Shanahan. *Solving the Frame Problem*, chapter 1. The MIT Press, 1997.
- Chin-Liang Chang and Richard Char-Tung Lee. *Symbolic Logic and Mechanical Theorem Proving*, chapters 2 and 3. Academic Press, 1973.

## Classical Planning

- restricted state-transition system $\Sigma=(S,A,\gamma)$
- planning problem $\mathcal{P}=(\Sigma,s_i,S_g)$

- Why study classical planning?
  - good for illustration purposes
  - algorithms that scale up reasonably well are known
  - extensions to more realistic models known
- What are the main issues?
  - how to represent states and actions
  - how to perform the solution search



## Planning as Theorem Proving

- idea:
  - represent states and actions in first-order predicate logic
  - prove that there is a state *s*
    - that is reachable from the initial state and
    - in which the goal is satisfied.
  - extract plan from proof

## Overview

- Propositional Logic
- First-Order Predicate Logic
- Representing Actions
- The Frame Problem
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## Propositions

- proposition: a declarative sentence (or statement) that can either *true* or *false*
- examples:
  - the robot is at location1
  - the crane is holding a container
- atomic propositions (atoms):
  - have no internal structure
  - notation: capital letters, e.g. P, Q, R, …

## Well-Formed Formulas

- an atom is a formula
- if G is a formula, then $(\neg G)$ is a formula
- if G and H are formulas, then $(G \land H)$, $(G \lor H)$, $(G \rightarrow H)$, $(G \leftrightarrow H)$ are formulas.
- all formulas are generated by applying the above rules

- logical connectives: $\neg$, $\land$, $\lor$, $\rightarrow$, $\leftrightarrow$



## Truth Tables

| G | H | $\neg$G | G$\land$H | G$\lor$H | G$\rightarrow$H | G$\leftrightarrow$H |
|---|---|---|---|---|---|---|
| *true* | *true* | *false* | *true* | *true* | *true* | *true* |
| *true* | *false* | *false* | *false* | *true* | *false* | *false* |
| *false* | *true* | *true* | *false* | *true* | *true* | *false* |
| *false* | *false* | *true* | *false* | *false* | *true* | *true* |

## Interpretations

- Let G be a propositional formula containing atoms $A_1,\ldots,A_n$.
- An interpretation *I* is an assignment of truth values to these atoms, i.e. *I*: $\{A_1,\ldots,A_n\} \rightarrow \{$*true*, *false*$\}$
- example:
  - formula G: $(P \wedge Q) \rightarrow (R \leftrightarrow (\neg S))$
  - interpretation *I*: P→*false*, Q→*true*, R→*true*, S→*true*
  - G evaluates to *true* under *I*: *I*(G) = *true*



## Validity and Inconsistency

- A formula is valid if and only if it evaluates to *true* under all possible interpretations.
- A formula that is not valid is invalid.
- A formula is inconsistent (or unsatisfiable) if and only if it evaluates to *false* under all possible interpretations.
- A formula that is not inconsistent is consistent (or satisfiable).
- examples:
  - valid: $P \vee \neg P$, $P \wedge (P \rightarrow Q) \rightarrow Q$
  - satisfiable: $(P \wedge Q) \rightarrow (R \leftrightarrow (\neg S))$
  - inconsistent: $P \wedge \neg P$

## Propositional Theorem Proving

- Problem: Given a set of propositional formulas $F_1 \ldots F_n$, decide whether
  - their conjunction $F_1 \wedge \ldots \wedge F_n$ is valid or satisfiable or inconsistent or
  - a formula G follows from (axioms) $F_1 \wedge \ldots \wedge F_n$, denoted $F_1 \wedge \ldots \wedge F_n \models G$
- decidable
- NP-complete, but relatively efficient algorithms known (for propositional logic)



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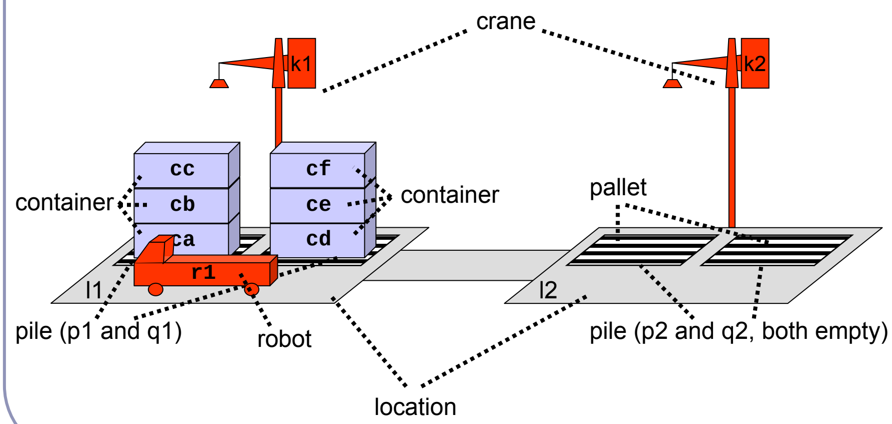
## First-Order Atoms

- objects are denoted by terms
  - constant terms: symbols denoting specific individuals
    - examples: loc1, loc2, …, robot1, robot2, …
  - variable terms: symbols denoting undefined individuals
    - examples: *l*,*l'*
  - function terms: expressions denoting individuals
    - examples: 1+3, father(john), father(mother(*x*))
- first-order propositions (atoms) state a relation between some objects
  - examples: adjacent(*l*,*l'*), occupied(*l*), at(*r*,*l*), …



## DWR Example State



## Objects in the DWR Domain

- locations {**loc1**, **loc2**, …}:
  - storage area, dock, docked ship, or parking or passing area
- robots {**robot1**, **robot2**, …}:
  - container carrier carts for one container
  - can move between adjacent locations
- cranes {**crane1**, **crane2**, …}:
  - belongs to a single location
  - can move containers between robots and piles at same location
- piles {**pile1**, **pile2**, …}:
  - attached to a single location
  - pallet at the bottom, possibly with containers stacked on top of it
- containers {**cont1**, **cont2**, …}:
  - stacked in some pile on some pallet, loaded onto robot, or held by crane
- **pallet**:
  - at the bottom of a pile



## Topology in the DWR Domain

- **adjacent(*l*,*l′*)**:
  location ***l*** is adjacent to location ***l′***
- **attached(*p*,*l*)**:
  pile ***p*** is attached to location ***l***
- **belong(*k*,*l*)**:
  crane ***k*** belongs to location ***l***

- topology does not change over time!

## Relations in the DWR Domain (1)

- **`occupied(`*`l`*`)`**:
  location ***l*** is currently occupied by a robot
- **`at(`*`r`*`,`*`l`*`)`**:
  robot ***r*** is currently at location ***l***
- **`loaded(`*`r`*`,`*`c`*`)`**:
  robot ***r*** is currently loaded with container ***c***
- **`unloaded(`*`r`*`)`**:
  robot ***r*** is currently not loaded with a container



## Relations in the DWR Domain (2)

- **`holding(`*`k`*`,`*`c`*`)`**:
  crane ***k*** is currently holding container ***c***
- **`empty(`*`k`*`)`**:
  crane ***k*** is currently not holding a container
- **`in(`*`c`*`,`*`p`*`)`**:
  container ***c*** is currently in pile ***p***
- **`on(`*`c`*`,`*`c′`*`)`**:
  container ***c*** is currently on container/pallet ***c′***
- **`top(`*`c`*`,`*`p`*`)`**:
  container/pallet ***c*** is currently at the top of pile ***p***

## Well-Formed Formulas

- an atom (relation over terms) is a formula
- if G and H are formulas, then $(\neg G)$ $(G \wedge H)$, $(G \vee H)$, $(G \rightarrow H)$, $(G \leftrightarrow H)$ are formulas
- if F is a formula and $x$ is a variable then $(\exists x\ F(x))$ and $(\forall x\ F(x))$ are formulas
- all formulas are generated by applying the above rules



## Formulas: DWR Examples

- adjacency is symmetric:
  $\forall l, l'\ \mathtt{adjacent}(l, l') \leftrightarrow \mathtt{adjacent}(l', l)$
- objects (robots) can only be in one place:
  $\forall r, l, l'\ \mathtt{at}(r, l) \wedge \mathtt{at}(r, l') \rightarrow l = l'$
- cranes are empty or they hold a container:
  $\forall k\ \mathtt{empty}(k) \vee \exists c\ \mathtt{holding}(k, c)$

## Semantics of First-Order Logic

- an interpretation *I* over a domain *D* maps:
  - each constant c to an element in the domain: $I(c) \in D$
  - each *n*-place function symbol f to a mapping: $I(f) \in D^n \rightarrow D$
  - each *n*-place relation symbol R to a mapping: $I(R) \in D^n \rightarrow \{true, false\}$
- truth tables for connectives ($\neg, \wedge, \vee, \rightarrow, \leftrightarrow$) as for propositional logic
- $I((\exists x\, F(x))) = true$ if and only if for at least one object $c \in D$: $I(F(c)) = true$.
- $I((\forall x\, F(x))) = true$ if and only if for every object $c \in D$: $I(F(c)) = true$.



## Theorem Proving in First-Order Logic

- F is valid: F is *true* under all interpretations
- F is inconsistent: F is *false* under all interpretations
- theorem proving problem (as before):
  - $F_1 \wedge \ldots \wedge F_n$ is valid / satisfiable / inconsistent or
  - $F_1 \wedge \ldots \wedge F_n \models G$
- semi-decidable
- resolution constitutes significant progress in mid-60s

## Substitutions

- replace a variable in an atom by a term
- example:
  - substitution: $\sigma = \{x \leftarrow 4, y \leftarrow f(5)\}$
  - atom A: greater($x$, $y$)
  - $\sigma(F)$ = greater(4, f(5))
- simple inference rule:
  - if $\sigma = \{x \leftarrow c\}$ and $(\forall x\ F(x)) \models F(c)$
  - example: $\forall x$ mortal($x$) $\models$ mortal(Confucius)



## Unification

- Let $A(t_1,\ldots,t_n)$ and $A(t'_1,\ldots,t'_n)$ be atoms.
- A substitution $\sigma$ is a unifier for $A(t_1,\ldots,t_n)$ and $A(t'_1,\ldots,t'_n)$ if and only if:
  $\sigma(A(t_1,\ldots,t_n)) = \sigma(A(t'_1,\ldots,t'_n))$
- examples:
  - P($x$, 2) and P(3, $y$) – unifier: $\{x \leftarrow 3, y \leftarrow 2\}$
  - P($x$, f($x$)) and P($y$, f($y$)) – unifiers: $\{x \leftarrow 3, y \leftarrow 3\}$, $\{x \leftarrow y\}$
  - P($x$, 2) and P($x$, 3) – no unifier exists

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## Representing States

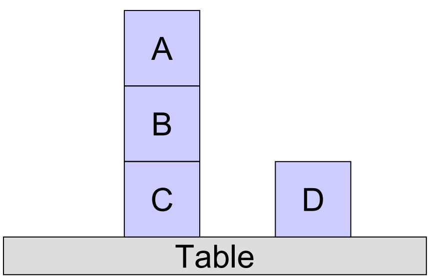
- represent domain objects as constants
  - examples: loc1, loc2, …, robot1, robot2, …
- represent relations as predicates
  - examples: adjacent(*l*,*l'*), occupied(*l*), at(*r*,*l*), …
- problem: truth value of some relations changes from state to state
  - examples: occupied(loc1), at(robot1,loc1)

## Situations and Fluents

- solution: make state explicit in representation through <u>situation</u> term
  - add situation parameter to changing relations:
    - occupied(loc1,s): location1 is occupied in situation s
    - at(robot1,loc1,s): robot1 is at location1 in situation s
  - or introduce predicate holds(*f*,*s*):
    - holds(occupied(loc1),s): location1 is occupied holds in situation s
    - holds(at(robot1,loc1),s): robot1 is at location1 holds in situation s
- <u>fluent</u>: a term or formula containing a situation term



## The Blocks World: Initial Situation




- $\Sigma_{si}$=
  on(C,Table,si) ∧
  on(B,C,si) ∧
  on(A,B,si) ∧
  on(D,Table,si) ∧
  clear(A,si) ∧
  clear(D,si) ∧
  clear(Table,si)

## Actions

- actions are non-tangible objects in the domain denoted by function terms
  - example: move(robot1,loc1,loc2): move robot1 from location loc1 to location loc2
- definition of an action through
  - a set of formulas defining applicability conditions
  - a set of formulas defining changes in the state brought about by the action

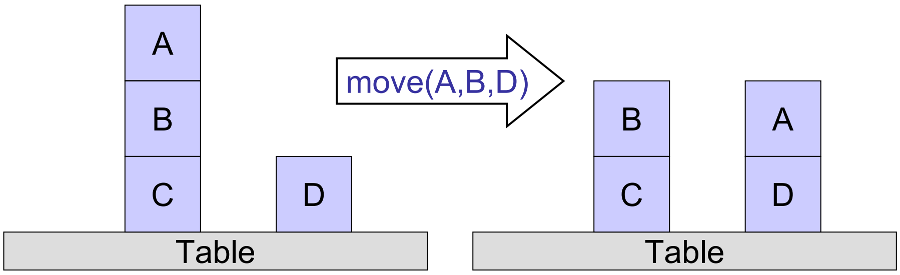


## Blocks World: Applicability

- $\Delta_a$=

$$
\begin{aligned}
\forall x,y,z,s:\ \text{applicable}(\text{move}(x,y,z),s) \leftrightarrow\ & \text{clear}(x,s) \land \\
& \text{clear}(z,s) \land \\
& \text{on}(x,y,s) \land \\
& x \neq \text{Table} \land \\
& x \neq z \land \\
& y \neq z
\end{aligned}
$$

## Blocks World: move Action




- single action move($x$,$y$,$z$): moving block $x$ from $y$ (where it currently is) onto $z$



## Applicability of Actions

- for each action specify applicability axioms of the form: ∀*params*,*s*: applicable(*action*(*params*),*s*) ↔ *preconds*(*params*,*s*)
- where:
  - "applicable" is a new predicate relating actions to states
  - *params* is a set of variables denoting objects
  - *action*(*params*) is a function term denoting an action over some objects
  - *preconds*(*params*) is a formula that is true iff *action*(*params*) can be performed in s

## Effects of Actions

- for each action specify effect axioms of the form:
  $\forall$*params*,*s*: applicable(*action*(*params*),*s*) $\rightarrow$ *effects*(*params*,result(*action*(*params*),*s*))
- where:
  - "result" is a new function that denotes the state that is the result of applying *action*(*params*) in *s*
  - *effects*(*params*,result(*action*(*params*),*s*)) is a formula that is true in the state denoted by result(*action*(*params*),*s*)



## Blocks World: Effect Axioms

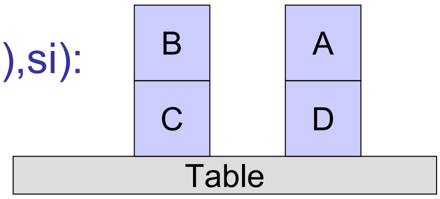
- $\Delta_e$=

  $\forall x,y,z,s$: applicable(move($x,y,z$),$s$) $\rightarrow$ on($x,z$,result(move($x,y,z$),$s$)) $\wedge$

  $\forall x,y,z,s$: applicable(move($x,y,z$),$s$) $\rightarrow$ clear($y$,result(move($x,y,z$),$s$))

## Blocks World: Derivable Facts

result(move(A,B,D),si):




- $\Sigma_{si} \wedge \Delta_a \wedge \Delta_e \models on(A,D,result(move(A,B,D),si))$
- $\Sigma_{si} \wedge \Delta_a \wedge \Delta_e \models clear(B,result(move(A,B,D),si))$



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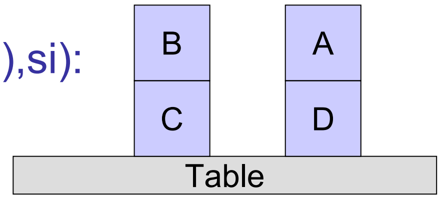
## Blocks World: Non-Derivable Fact

result(move(A,B,D),si):




- not derivable:
  $\Sigma_{si} \wedge \Delta_a \wedge \Delta_e \models$
  on(B,C,result(move(A,B,D),si))



## The Non-Effects of Actions

- effect axioms describe what changes when an action is applied, but not what does not change
- example: move robot
  - does not change the colour of the robot
  - does not change the size of the robot
  - does not change the political system in the UK
  - does not change the laws of physics

## Frame Axioms

- for each action and each fluent specify a <u>frame axiom</u> of the form:

  $\forall params,vars,s$: $fluent(vars,s) \wedge params \neq vars \rightarrow$
  $fluent(vars,\mathrm{result}(action(params),s))$

- where:
  - $fluent(vars,s)$ is a relation that is not affected by the application of the action
  - $params \neq vars$ is a conjunction of inequalities that must hold for the action to not effect the fluent



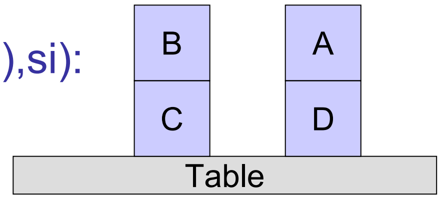
## Blocks World: Frame Axioms

- $\Delta_f$=

  $\forall v,w,x,y,z,s$: $\mathrm{on}(v,w,s) \wedge v \neq x \rightarrow$
  $\mathrm{on}(v,w,\mathrm{result}(\mathrm{move}(x,y,z),s)) \wedge$
  $\forall v,w,x,y,z,s$: $\mathrm{clear}(v,s) \wedge v \neq z \rightarrow$
  $\mathrm{clear}(v,\mathrm{result}(\mathrm{move}(x,y,z),s))$

## Blocks World: Derivable Fact with Frame Axioms

result(move(A,B,D),si):




- now derivable:
  $\Sigma_{si} \wedge \Delta_a \wedge \Delta_e \wedge \Delta_f \models$
  on(B,C,result(move(A,B,D),si))



## Coloured Blocks World

- like blocks world, but blocks have colour (new fluent) and can be painted (new action)
- new information about si:
  - $\forall x$: colour($x$,Blue,si))
- new effect axiom:
  - $\forall x,y,s$: colour($x$,$y$,result(paint($x$,$y$),$s$))
- new frame axioms:
  - $\forall v,w,x,y,z,s$: colour($v$,$w$,$s$) $\rightarrow$ colour($v$,$w$,result(move($x$,$y$,$z$),$s$))
  - $\forall v,w,x,y,s$: colour($v$,$w$,$s$) $\wedge$ $v \neq x$ $\rightarrow$ colour($v$,$w$,result(paint($x$,$y$),$s$))
  - $\forall v,w,x,y,s$: on($v$,$w$,$s$) $\rightarrow$ on($v$,$w$,result(paint($x$,$y$),$s$))
  - $\forall v,w,x,y,s$: clear($v$,$w$,$s$) $\rightarrow$ clear($v$,$w$,result(paint($x$,$y$),$s$))

## The Frame Problem

- problem: need to represent a long list of facts that are not changed by an action
- the frame problem:
  - construct a formal framework
  - for reasoning about actions and change
  - in which the non-effects of actions do not have to be enumerated explicitly



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## Approaches to the Frame Problem

- use a different style of representation in first-order logic (same formalism)
- use a different logical formalism, e.g. non-monotonic logic
- write a procedure that generates the right conclusions and forget about the frame problem



## Criteria for a Solution

- representational parsimony:
  representation of the effects of actions should be compact
- expressive flexibility:
  representation suitable for domains with more complex features
- elaboration tolerance:
  effort required to add new information is proportional to the complexity of that information

## The Universal Frame Axiom

- frame axiom for all actions, fluents, and situations:
  $\forall a,f,s$: holds($f$,$s$) ∧ ¬affects($a$,$f$,$s$) → holds($f$,result(a,$s$))
- where "affects" is a new predicate that relates actions, fluents, and situations
- ¬affects($a$,$f$,$s$) is true if and only if the action $a$ does not change the value of the fluent $f$ in situation $s$



## Coloured Blocks World Example Revisited

- coloured blocks world new frame axioms:
  - $\forall v,w,x,y,z,s$: $x \neq v$ → ¬affects(move($x$,$y$,$z$), on($v$,$w$), $s$)
  - $\forall v,w,x,y,s$: ¬affects(paint($x$,$y$), on($v$,$w$), $s$)
  - $\forall v,x,y,z,s$: $y \neq v$ ∧ $z \neq v$ → ¬affects(move($x$,$y$,$z$), clear($v$), $s$)
  - $\forall v,x,y,s$: ¬affects(paint($x$,$y$), clear($v$), $s$)
  - $\forall v,w,x,y,z,s$: ¬affects(move($x$,$y$,$z$), colour($v$,$w$), $s$)
  - $\forall v,w,x,y,s$: $x \neq v$ → ¬affects(paint($x$,$y$), colour($v$,$w$), $s$)
- more compact, but not fewer frame axioms

## Explanation Closure Axioms

- idea: infer the action from the affected fluent:
  - $\forall a,v,w,s$: affects($a$, on($v$,$w$), $s$) → $\exists x,y$: a=move($v$,$x$,$y$)
  - $\forall a,v,s$: affects($a$, clear($v$), $s$) →
    ($\exists x,z$: a=move($x$,$v$,$z$)) ∨ ($\exists x,y$: a=move($x$,$y$,$v$))
  - $\forall a,v,w,s$: affects($a$, colour($v$,$w$), $s$) → $\exists x$: a=paint($v$,$x$)
- allows to draw all the desired conclusions
- reduces the number of required frame axioms
- also allows to the draw the conclusion:
  - $\forall a,v,w,x,y,s$: a≠move($v$,$x$,$y$) → ¬affects($a$, on($v$,$w$), $s$)



## The Limits of Classical Logic

- monotonic consequence relation:
  $\Delta \models \phi$ implies $\Delta \wedge \delta \models \phi$
- problem:
  - need to infer when a fluent is not affected by an action
  - want to be able to add actions that affect existing fluents
- monotonicity: if ¬affects($a$, $f$, $s$) holds in a theory it must also hold in any extension

## Using Non-Monotonic Logics

- non-monotonic logics rely on default reasoning:
  - jumping to conclusions in the absence of information to the contrary
  - conclusions are assumed to be true by default
  - additional information may invalidate them
- application to frame problem:
  - explanation closure axioms are default knowledge
  - effect axioms are certain knowledge



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