# Scheduling

Planning with Actions that Require Resources

## Literature

- Malik Ghallab, Dana Nau, and Paolo Traverso. *Automated Planning – Theory and Practice*, chapter 15. Elsevier/Morgan Kaufmann, 2004.
- Michael Pinedo. *Scheduling: Theory, Algorithms and Systems*, Prentice Hall, 2001.
- Peter Brucker. *Scheduling Algorithms*, Springer Verlag, 2004.

## Planning and Scheduling

- solution to planning problem:
  - plan: partially ordered set of actions
  - actions: fully instantiated operators
    - require resources
- resources:
  - can be modelled as parameters of an action
    - problem: planning algorithms tries out all possibilities (inefficient)
  - alternative approach:
    - allow unbound resource variables in plan (planning)
    - find assignment of resources to actions (scheduling)



## Overview

- Scheduling Problems and Schedules
- ➨ Searching for Schedules

## Actions and Resources

- resources: an entity needed to perform an action
  - state variables: modified by actions in absolute ways
    - example: move(*r,l.l'*):
    - location changes from *l* to *l'*
  - resource variables: modified by actions in relative ways
    - example: move(*r,l.l'*):
    - fuel level changes from *f* to *f-f'*



## Actions with Time Constraints

- Let *a* be an action in a planning domain:
  - attached time constraints:
    - earliest start time: $s_{min}(a)$ – actual start time: $s(a)$
    - latest end time: $s_{max}(a)$ – actual end time: $e(a)$
    - duration: $d(a)$
- action types:
  - preemptive actions: cannot be interrupted
    - $d(a) = e(a) - s(a)$
  - non-preemptive actions: can be interrupted
    - resources available to other actions during interruption

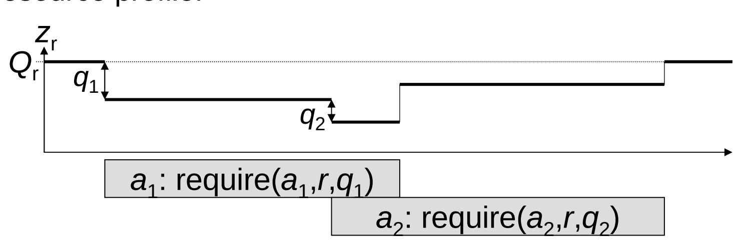
## Actions with Resource Constraints

- Let $a$ be an action in a planning domain:
  - attached resource constraints:
    - required resource: $r$
    - quantity of resource required: $q$
  - reusable: resource will be available to other actions after this action is completed
  - consumable: resource will be consumed when action is complete



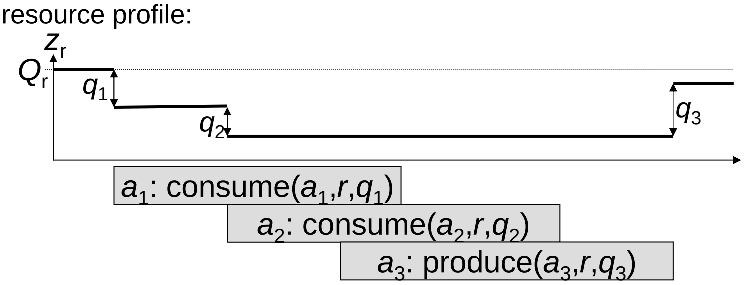
## Reusable Resources

- resource availability:
  - total capacity: $Q_r$
  - current level at time $t$: $z_r(t)$
- resource requirements:
  - require($a$,$r$,$q$): action $a$ requires $q$ units of resource $r$ while it is active
- resource profile:



## Consumable Resources

- resource availability:
  - total reservoir at $t_0$: $Q_r$
  - current level at time $t$: $z_r(t)$
- resource consumption/production:
  - consume($a,r,q$): action $a$ requires $q$ units of resource $r$
  - produce($a,r,q$): action $a$ produces $q$ units of resource $r$
- resource profile:

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## Other Resource Features

- discrete vs. continuous
  - countable number of units: cranes, bolts
  - real-valued amount: bandwidth, electricity
- unary
  - $Q_r$=1; exactly one resource of this type available
- sharable
  - can be used by several actions at the same time
- resources with states
  - actions may require resources in specific state

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## Combining Resource Constraints

- conjunction:
  - action uses multiple resources while being performed
- disjunction:
  - action requires resources as alternatives
  - cost/time may depend on resource used
- resource types:
  - resource-class($s$) = $\{r_1,\ldots,r_m\}$: require($a,s,q$)
  - equivalent to disjunction over identical resources



## Cost Functions and Optimization Criteria

- cost function parameters
  - quantity of resource required
  - duration of requirement
- optimization criteria:
  - total schedule cost
  - makespan (end time of last action)
  - weighted completion time
  - (weighted) number of late actions
  - (weighted) maximum tardiness
  - resource usage

## Machine Scheduling

- machine: resource of unit capacity
  - either available or not available at time $t$
  - cannot process two actions at the same time
- job $j$: partially ordered set of actions $a_{j1},\ldots,a_{jk}$
  - action $a_{ji}$ requires
    - one resource type
    - for a number of time units
  - actions in same job must be processed sequentially
  - actions in different jobs are independent (not ordered)
- machine scheduling problem:
  - given: $n$ jobs and $m$ machines
  - schedule: mapping from actions to machines + start times

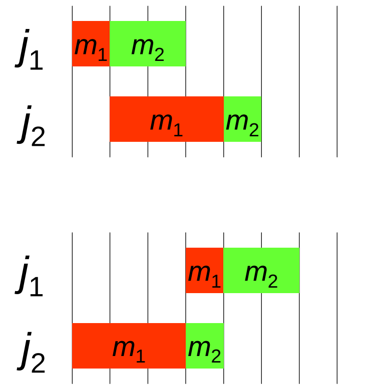


## Example: Scheduling Problem

- machines:
  - $m_1$ of resource type $r_1$
  - $m_2$, $m_3$ of resource type $r_2$
- jobs:
  - $j_1$: $\langle r_1(3), r_2(3), r_1(3)\rangle$
    - three actions, totally ordered
    - $a_{11}$ requires 3 units of resource type 1, etc.
  - $j_2$: $\langle r_2(3), r_1(5)\rangle$
  - $j_3$: $\langle r_1(3), r_1(2), r_2(3), r_1(5)\rangle$

## Example: Schedules by Job

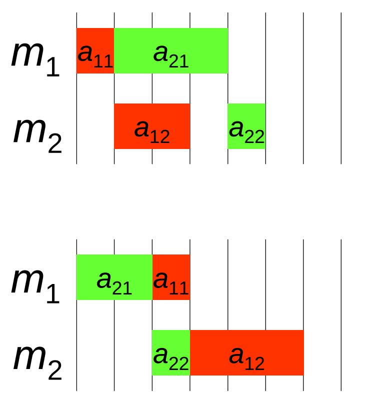
- machines:
  - $m_1$ of type $r_1$
  - $m_2$ of type $r_2$
- jobs:
  - $j_1$: $\langle r_1(1), r_2(2)\rangle$
  - $j_2$: $\langle r_1(3), r_2(1)\rangle$






## Example: Schedules by Machine

- machines:
  - $m_1$ of type $r_1$
  - $m_2$ of type $r_2$
- jobs:
  - $j_1$: $\langle r_1(1), r_2(2)\rangle$
  - $j_2$: $\langle r_1(3), r_2(1)\rangle$



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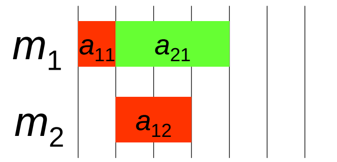
## Assignable Actions

- Let $P$ be a machine scheduling problem. Let $S$ be a partially defined schedule.
- An action $a_{ji}$ of some job $j_i$ in $P$ is <u>unassigned</u> if it does not appear in $S$.
- An action $a_{ji}$ of some job $j_i$ in $P$ is <u>assignable</u> if it has no unassigned predecessors in $S$.

## Example: Assignable Actions

- problem $P$:
  - machines:
    - $m_1$ of type $r_1$
    - $m_2$ of type $r_2$
  - jobs:
    - $j_1$: $\langle r_1(1), r_2(2)\rangle$
    - $j_2$: $\langle r_1(3), r_2(1)\rangle$
    - $j_2$: $\langle r_1(3), r_2(1), r_1(3)\rangle$
- partial schedule $S$:




  - unassigned:
    - $a_{22}$, $a_{31}$, $a_{32}$, $a_{33}$
  - assignable:
    - $a_{22}$, $a_{31}$



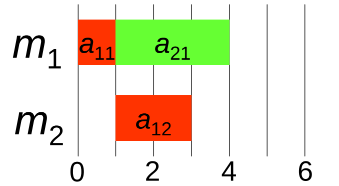
## Earliest Assignable Time

- Let $a_{ji}$ be an assignable action in $S$. The <u>earliest assignable time</u> for $a_{ji}$ on machine $m$ in $S$ is:
  - the end of the last action currently scheduled on $m$ in $S$, or
  - the end of the last predecessor ($a_{j0}$ … $a_{ji-1}$) in S,

  whichever comes later.

## Example: Earliest Assignable Time

- problem $P$:
  - machines:
    - $m_1$ of type $r_1$
    - $m_2$ of type $r_2$
  - jobs:
    - $j_1$: $\langle r_1(1), r_2(2)\rangle$
    - $j_2$: $\langle r_1(3), r_2(1)\rangle$
    - $j_2$: $\langle r_1(3), r_2(1), r_1(3)\rangle$
- partial schedule $S$:




  - earliest assignable time for $a_{22}$ on $m_2$: 4
  - earliest assignable time for $a_{31}$ on $m_1$:4



## Heuristic Search

heuristicScheduler($P$,$S$)
    *assignables* ← $P$.getAssignables($S$)
    **if** *assignables*.isEmpty() **then return** $S$
    *action* ← *assignables*.selectOne()
    *machines* ← $P$.getMachines(*action*)
    *machine* ← *machines*.selectOne()
    *time* ← $S$.getEarliestAssignableTime(*action*, *machine*)
    S ← S + assign(*action*, *machine*, *time*)
    **return** heuristicScheduler($P$,$S$)

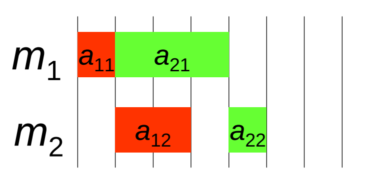
## Using Local Search

- issues:
  - representing schedules
  - generating a random initial schedule
  - generating neighbours
  - evaluating neighbours (schedules)



## Schedule Representation

- representation:
  - totally ordered list of all actions with assigned machines
  - example: $\langle(a_{11},m_1), (a_{21},m_1), (a_{12},m_2), (a_{22},m_2)\rangle$
- schedule:
  - assign actions in sequence to given machines at earliest assignable times
  - example:



## Initial Schedule and Evaluation

- generating random schedules:
  - randomly choose an assignable action
  - randomly choose a machine of the right resource type for that action
  - append the action-machine pair to the list of assignments
  - do this until all actions are assigned
- evaluating schedules:
  - generate schedule from list
  - apply optimization criterion



## Generating Neighbours

- machine neighbours:
  - change the machine assigned to an action to any other machine
- position neighbours:
  - change the position of an action *a* in the list:
    - $a_{min}$:the latest predecessor of *a* in the current list
    - $a_{max}$:the earliest successor of *a* in the current list
    - move *a* anywhere between $a_{min}$ and $a_{max}$

## LocalSearchScheduler: Pseudo Code

```
function LocalSearchScheduler(P)
best ← randomSchedule(P)
loop MAXLOOP times
    S ← randomSchedule(P)
    do
        succs ← S.getBestNeighbours(P)
        next ← succs.selectOne()
        if S.evaluate() < next.evaluate() then
            S ← next
    while S = next
    if S.evaluate() > best.evaluate() then
        best ← S
return best
```



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