Plan-Space Search

Searching for a Solution Plan in a Graph of Partial Plans

Literature

- Malik Ghallab, Dana Nau, and Paolo Traverso. *Automated Planning – Theory and Practice*, chapter 2 and 5. Elsevier/Morgan Kaufmann, 2004.
- J. Penberthy and D. S. Weld. UCPOP: A sound, complete, partial-order for ADL. In *Proceeding s of the International Conference on Knowledge Representation and Reasoning*, pages 103-114, 1992.

State-Space vs. Plan-Space Search

- state-space search: search through graph of nodes representing world states
- plan-space search: search through graph of partial plans
  - nodes: partially specified plans
  - arcs: plan refinement operations
  - solutions: partial-order plans

Overview

- The Search Space of Partial Plans
- Plan-Space Search Algorithms
- Extensions of the STRIPS Representation

Partial Plans

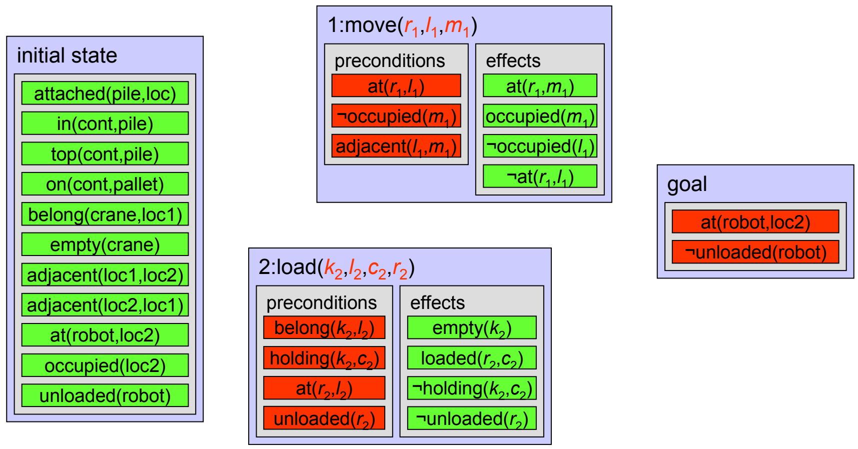
- plan: set of actions organized into some structure
- partial plan:
  - subset of the actions
  - subset of the organizational structure
    - temporal ordering of actions
    - rationale: what the action achieves in the plan
  - subset of variable bindings

Adding Actions

- partial plan contains actions
  - initial state
  - goal conditions
  - set of operators with different variables

- reason for adding new actions
  - to achieve unsatisfied preconditions
  - to achieve unsatisfied goal conditions

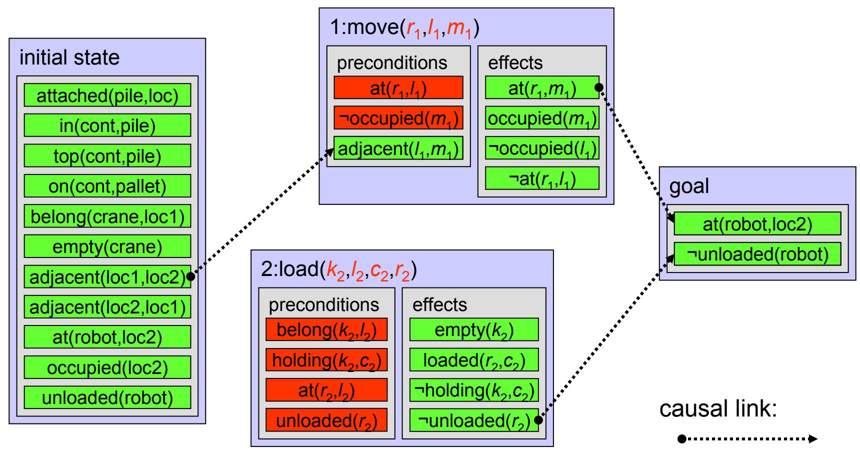
Adding Actions: Example



Adding Causal Links

- partial plan contains causal links
  - links from the provider
    - an effect of an action or
    - an atom that holds in the initial state
  - to the consumer
    - a precondition of an action or
    - a goal condition
- reasons for adding causal links
  - prevent interference with other actions

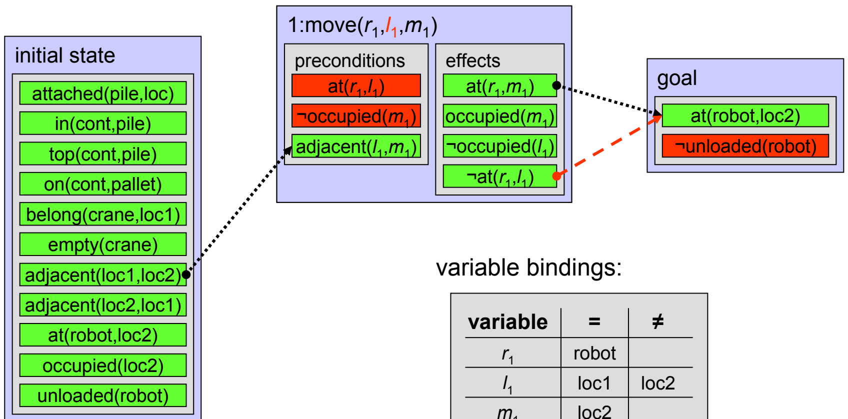
Adding Causal Links: Example



Adding Variable Bindings

- partial plan contains variable bindings
  - new operators introduce new (copies of) variables into the plan
  - solution plan must contain actions
  - variable binding constraints keep track of possible values for variables and co-designation
- reasons for adding variable bindings
  - to turn operators into actions
  - to unify and effect with the precondition it supports

Adding Variable Bindings: Example



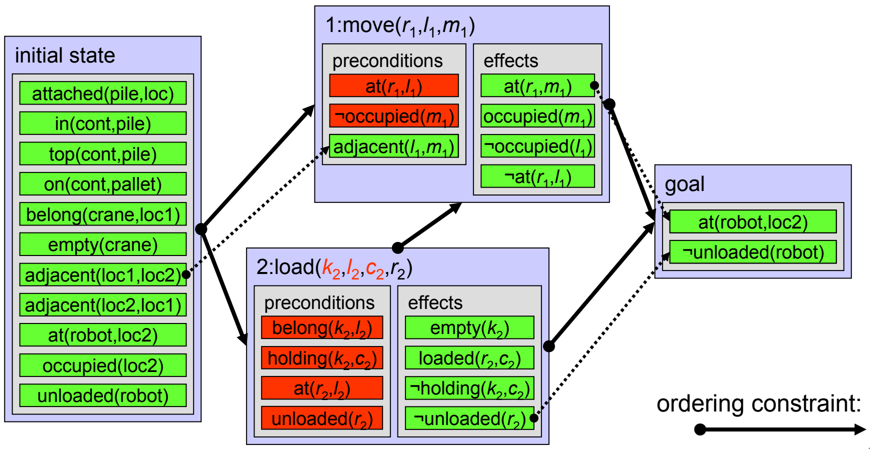
variable bindings:

variable	=	≠
r_1	robot	
l_1	loc1	loc2
m_1	loc2	

Adding Ordering Constraints

- partial plan contains ordering constraints
  - binary relation specifying the temporal order between actions in the plan
- reasons for adding ordering constraints
  - all actions after initial state
  - all actions before goal
  - causal link implies ordering constraint
  - to avoid possible interference

Adding Ordering Constraints: Example



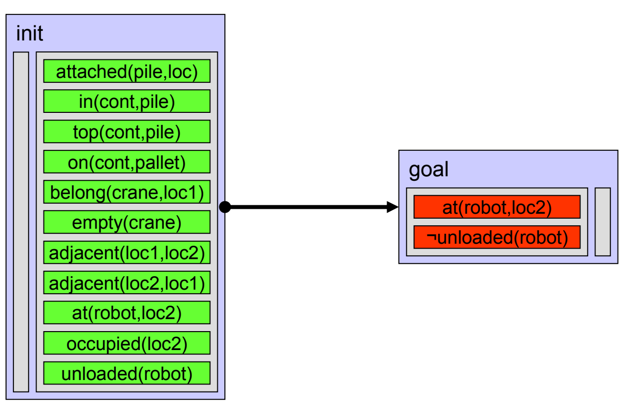
Definition of Partial Plans

- A partial plan is a tuple $\pi = (A, \prec, B, L)$, where:
  - $A = \{a_1, \ldots, a_k\}$ is a set of partially instantiated planning operators;
  - $\prec$ is a set of ordering constraints on A of the form $(a_i \prec a_j)$;
  - B is a set of binding constraints on the variables of actions in A of the form $x=y$, $x \neq y$, or $x \in D_x$;
  - L is a set of causal links of the form $\langle a_i -[p] \rightarrow a_j \rangle$ such that:
    - a_i and a_j are actions in A;
    - the constraint $(a_i \prec a_j)$ is in $\prec$;
    - proposition p is an effect of a_i and a precondition of a_j; and
    - the binding constraints for variables in a_i and a_j appearing in p are in B.

Plan-Space Search: Initial Search State

- represent initial state and goal as dummy actions
  - init: no preconditions, initial state as effects
  - goal: goal conditions as preconditions, no effects
- empty plan π_0 = ({init, goal},{(init≺goal)},{},{}):
  - two dummy actions init and goal;
  - one ordering constraint: init before goal;
  - no variable bindings; and
  - no causal links.

Plan-Space Search: Initial Search State Example



Plan-Space Search: Successor Function

- states are partial plans
- generate successor through plan refinement operators (one or more):
  - adding an action to *A*
  - adding an ordering constraint to $\prec$
  - adding a binding constraint to *B*
  - adding a causal link to *L*

Total vs. Partial Order

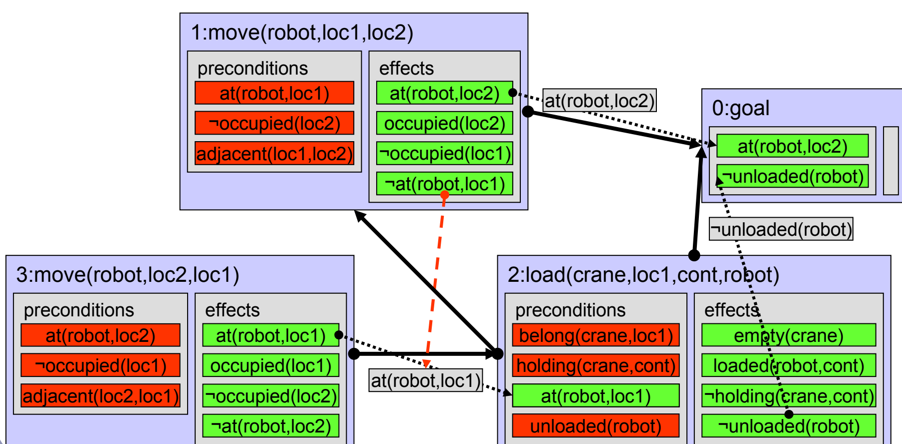
- Let $\mathcal{P}=(\Sigma,s_i,g)$ be a planning problem. A plan π is a solution for $\mathcal{P}$ if $\gamma(s_i,\pi)$ satisfies g.

- problem: $\gamma(s_i,\pi)$ only defined for sequence of ground actions
  - partial order corresponds to total order in which all partial order constraints are respected
  - partial instantiation corresponds to grounding in which variables are assigned values consistent with binding constraints

Partial Order Solutions

- Let $\mathcal{P}=(\Sigma,s_i,g)$ be a planning problem. A plan $\pi = (A,\prec,B,L)$ is a (partial order) solution for $\mathcal{P}$ if:
  - its ordering constraints $\prec$ and binding constraints B are consistent; and
  - for every sequence $\langle a_1,\ldots,a_k\rangle$ of all the actions in A-{init, goal} that is
    - totally ordered and grounded and respects $\prec$ and B
    - $\gamma(s_i, \langle a_1,\ldots,a_k\rangle)$ must satisfy g.

Threat: Example



Threats

- An action a_k in a partial plan π = $(A,\prec,B,L)$ is a threat to a causal link $\langle a_i -[p]\rightarrow a_j \rangle$ iff:
  - a_k has an effect $\neg q$ that is possibly inconsistent with p, i.e. q and p are unifiable;
  - the ordering constraints $(a_i \prec a_k)$ and $(a_k \prec a_j)$ are consistent with $\prec$; and
  - the binding constraints for the unification of q and p are consistent with B.

Flaws

- A flaw in a plan π = $(A,\prec,B,L)$ is either:
  - an unsatisfied sub-goal, i.e. a precondition of an action in A without a causal link that supports it; or
  - a threat, i.e. an action that may interfere with a causal link.

Flawless Plans and Solutions

- **Proposition**: A partial plan $\pi = (A,\prec,B,L)$ is a solution to the planning problem $\mathcal{P}=(\Sigma,s_i,g)$ if:
  - π has no flaw;
  - the ordering constraints $\prec$ are not circular; and
  - the variable bindings B are consistent.
- Proof: by induction on number of actions in A
  - base case: empty plan
  - induction step: totally ordered plan minus first step is solution implies plan including first step is a solution: $\gamma(s_i, \langle a_1,\ldots,a_k\rangle) = \gamma(\gamma(s_i, a_1), \langle a_2,\ldots,a_k\rangle)$

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- ➡ Plan-Space Search Algorithms
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Plan-Space Planning as a Search Problem

- given: statement of a planning problem $P=(O,s_i,g)$
- define the search problem as follows:
  - initial state: π_0 = ({init, goal},{(init≺goal)},{},{})
  - goal test for plan state *p*: *p* has no flaws
  - path cost function for plan π: $|\pi|$
  - successor function for plan state *p*: refinements of *p* that maintain ≺ and *B*

PSP Procedure: Basic Operations

- PSP: Plan-Space Planner
- main principle: refine partial π plan while maintaining ≺ and *B* consistent until π has no more flaws
- basic operations:
  - find the flaws of π, i.e. its sub-goals and its threats
  - select one of the flaws
  - find ways to resolve the chosen flaw
  - choose one of the resolvers for the flaw
  - refine π according to the chosen resolver

PSP: Pseudo Code

function PSP(*plan*)
  allFlaws ← *plan*.openGoals() + *plan*.threats()
  if *allFlaws*.empty() **then return** *plan*
  flaw ← *allFlaws*.selectOne()
  allResolvers ← *flaw*.getResolvers(*plan*)
  if *allResolvers*.empty() **then return** failure
  resolver ← *allResolvers*.chooseOne()
  newPlan ← *plan*.refine(*resolver*)
  return PSP(*newPlan*)

PSP: Choice Points

- *resolver* ← *allResolvers*.chooseOne()
  - non-deterministic choice
- *flaw* ← *allFlaws*.selectOne()
  - deterministic selection
  - all flaws need to be resolved before a plan becomes a solution
  - order not important for completeness
  - order is important for efficiency

Implementing *plan*.openGoals()

- finding unachieved sub-goals (incrementally):
  - in π_0: goal conditions
  - when adding an action: all preconditions are unachieved sub-goals
  - when adding a causal link: protected proposition is no longer unachieved

Implementing *plan*.threats()

- finding threats (incrementally):
  - in π_0: no threats
  - when adding an action a_{new} to $\pi = (A,\prec,B,L)$:
    - for every causal link $\langle a_i -[p]\rightarrow a_j\rangle \in L$

      if ($a_{new} \prec a_i$) or ($a_j \prec a_{new}$) then next link

      else for every effect q of a_{new}

      if ($\exists\sigma$: $\sigma(p)=\sigma(\neg q)$) then q of a_{new} threatens $\langle a_i -[p]\rightarrow a_j\rangle$
  - when adding a causal link $\langle a_i -[p]\rightarrow a_j\rangle$ to $\pi = (A,\prec,B,L)$:
    - for every action $a_{old} \in A$

      if ($a_{old} \prec a_i$) or ($a_j = a_{old}$) or ($a_j \prec a_{old}$) then next action

      else for every effect q of a_{old}

      if ($\exists\sigma$: $\sigma(p)=\sigma(\neg q)$) then q of a_{old} threatens $\langle a_i -[p]\rightarrow a_j\rangle$

Implementing *flaw*.getResolvers(*plan*)

- for unachieved precondition p of a_g:
  - add causal links to an existing action:
    - for every action $a_{old} \in A$
      if ($a_g = a_{old}$) or ($a_g \prec a_{old}$) then next action
      else for every effect q of a_{old}
      if ($\exists\sigma$: $\sigma(p)=\sigma(q)$) then adding
      $\langle a_{old} -[\sigma(p)] \rightarrow a_g \rangle$ is a resolver
  - add a new action and a causal link:
    - for every effect q of every operator o
      if ($\exists\sigma$: $\sigma(p)=\sigma(q)$) then adding
      a_{new}=o.newInstance() and
      $\langle a_{new} -[\sigma(p)] \rightarrow a_g \rangle$ is a resolver

Implementing *flaw*.getResolvers(*plan*)

- for effect q of action a_t threatening $\langle a_i -[p] \rightarrow a_j \rangle$:
  - order action before threatened link:
    - if ($a_t = a_i$) or ($a_j \prec a_t$) then not a resolver
      else adding ($a_t \prec a_i$) is a resolver
  - order threatened link before action:
    - if ($a_t = a_i$) or ($a_t \prec a_i$) then not a resolver
      else adding ($a_j \prec a_t$) is a resolver
  - extend variable bindings such that unification fails:
    - for every variable v in p or q
      if $v \neq \sigma(v)$ is consistent with B then
      adding $v \neq \sigma(v)$ is a resolver

Implementing *plan*.refine(*resolver*)

- refines partial plan with elements in resolver by adding:
  - an ordering constraint;
  - one or more binding constraints;
  - a causal link; and/or
  - a new action.
- no testing required
- must update flaws:
  - unachieved preconditions (see: *plan*.openGoals())
  - threats (see: *plan*.threats())

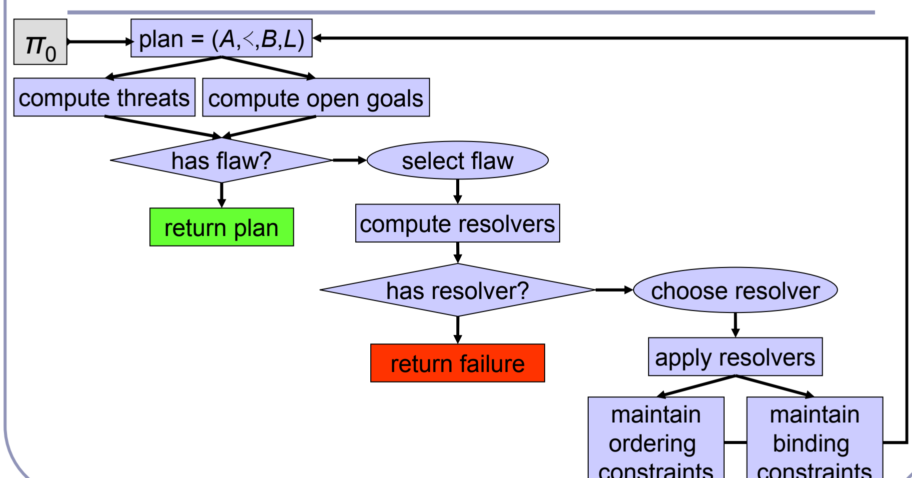
Maintaining Ordering Constraints

- required operations:
  - query whether $(a_i \prec a_j)$
  - adding $(a_i \prec a_j)$
- possible internal representations:
  - maintain set of predecessors/successors for each action as given
  - maintain only direct predecessors/successors for each action
  - maintain transitive closure of $\prec$ relation

Maintaining Variable Binding Constraints

- types of constraints:
  - unary constraints: $x \in D_x$
  - equality constraints: $x = y$
  - inequalities: $x \neq y$
- note: general CSP problem is NP-complete

PSP: Data Flow



PSP: Sound and Complete

- **Proposition**: The PSP procedure is sound and complete: whenever π_0 can be refined into a solution plan, PSP(π_0) returns such a plan.
- Proof:
  - soundness: ≺ and *B* are consistent at every stage of the refinement
  - completeness: induction on the number of actions in the solution plan

PSP Implementation: PoP

- extended input:
  - partial plan (as before)
  - agenda: set of pairs (*a*,*p*) where *a* is an action an *p* is one of its preconditions
- search control by flaw type
  - unachieved sub-goal (on agenda): as before
  - threats: resolved as part of the successor generation process

PoP: Pseudo Code (1)

function PoP(*plan*, *agenda*)
  if *agenda*.empty() **then return** *plan*
  (a_g,p_g) ← *agenda*.selectOne()
  agenda ← *agenda* - (a_g,p_g)
  relevant ← *plan*.getProviders(p_g)
  if *relevant*.empty() **then return** failure
  (a_p,p_p,σ) ← *relevant*.chooseOne()
  plan.L ← *plan.L* ∪ $\langle a_p -[p]\rightarrow a_g \rangle$
  plan.B ← *plan.B* ∪ σ

PoP: Pseudo Code (2)

  if $a_p \notin$ *plan.A* **then**
    plan.add(a_p)
    agenda ← *agenda* + a_p.*preconditions*
  newPlan ← *plan*
  for each *threat* on $\langle a_p -[p]\rightarrow a_g \rangle$ **or** due to a_p **do**
    allResolvers ← *threat*.getResolvers(*newPlan*)
    if *allResolvers*.empty() **then return** failure
    resolver ← *allResolvers*.chooseOne()
    newPlan ← *newPlan*.refine(*resolver*)
  return PSP(*newPlan,agenda)*

State-Space vs. Plan-Space Planning

- state-space planning
  - finite search space
  - explicit representation of intermediate states
  - action ordering reflects control strategy
  - causal structure only implicit
  - search nodes relatively simple and successors easy to compute
- plan-space planning
  - finite search space
  - no intermediate states
  - choice of actions and organization independent
  - explicit representation of rationale
  - search nodes are complex and successors expensive to compute

Using Partial-Order Plans: Main Advantages

- more flexible during execution
- using constraint managers facilitates extensions such as:
  - temporal constraints
  - resource constraints
- distributed and multi-agent planning fit naturally into the framework

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Existential Quantification in Goals

- allow existentially quantified conjunction of literals as goal:
  - $g = \exists x_1,\ldots,x_n: l_1 \wedge \ldots \wedge l_m$
- rewrite into equivalent planning problem:
  - new goal $g' = \{p\}$ where p is an unused proposition symbol
  - introduce additional operator $o = (\text{op-g}(x_1,\ldots,x_n),\{l_1,\ldots,l_m\},\{p\})$
- in plan-space search: no change needed

DWR Example: Existential Quantification in Goals

- goal: $\exists x,y$: on(x,c1) ∧ on(y,c2)
- rewritten goal: p
- new operator: o = (op-g(x,y),{on(x,c1),on(y,c2)},{p})
- plan-space search goal: on(x,c1) ∧ on(y,c2)

Typed Variables

- allow typed variables in operators:
  - name(o) = $n(x_1{:}t_1,\ldots,x_k{:}t_k)$ where t_i is the type of variable x_i
- rewrite into equivalent planning problem:
  - add preconditions $\{t_1(x_1),\ldots,t_k(x_k)\}$ to o
  - if constant c_i is of type t_j, add rigid relation $t_j(c_i)$ to the initial state
  - remove types from operator names

DWR Example: Typed Variables

- operator: move(r:robot,l:location,m:location)
  - precond: adjacent(l,m), at(r,l), ¬occupied(m)
  - effects: at(r,m), occupied(m), ¬occupied(l), ¬at(r,l)
- rewritten operator:move(r,l,m)
  - precond: adjacent(l,m), at(r,l), ¬occupied(m), robot(r), loaction(l), location(m)
  - effects: at(r,m), occupied(m), ¬occupied(l), ¬at(r,l)
- rewritten initial state:
  - s_i ∪ {robot(r1),container(c1),container(c2),…}

Conditional Operators

- conditional planning operators:
  - o = (n,(precond_0,effects_0),…,(precond_n,effects_n)) where:
    - $n = o(x_1,\ldots,x_n)$ as before,
    - (precond_0,effects_0) are the unconditional preconditions and effects of the operator, and
    - (precond_i,effects_i) for i≥1 are the conditional preconditions and effects of the operator.
  - a ground instance a of o is applicable in state s if s satisfies precond_0
  - let I={i∈[0,n] | s satisfies $\text{precond}_i(a)$}; then:
    - $\gamma(s,a)=(s - \cup_{(i\in I)}\text{effects}^-(a)) \cup (\cup_{(i\in I)}\text{effects}^+(a))$

DWR Example: Conditional Operators

- relation at(o,l): object o is at location l
- conditional move operator:
  move(r,l,m,c)
  - $precond_0$: adjacent(l,m), at(r,l), ¬occupied(m)
  - $effects_0$: at(r,m), occupied(m), ¬occupied(l), ¬at(r,l)
  - $precond_1$: loaded(r,c)
  - $effects_1$: at(c,m), ¬at(c,l)

Extending PoP to handle Conditional Operators

- modifying *plan*.getProviders(p_g):
  - new action with matching conditional effect
  - add precondition of conditional effect to agenda
- managing conditional threats:
  - new alternative resolver: add negated precondition of threatening conditional effect to agenda

Quantified Expressions

- allow universally quantified variables in conditional preconditions and effects:
  - for-all $x_1,\ldots,x_n$: (precond$_i$,effects$_i$)
- *a* is applicable in state *s* if *s* satisfies precond$_0$
- Let σ be a substitution for $x_1,\ldots,x_n$ such that σ(precond$_i$(*a*)) and σ(effects$_i$(*a*)) are ground.
  - If *s* satisfies σ(precond$_i$(*a*)) then
  - σ(effects$_i$(*a*)) are effects of the action.

DWR Example: Quantified Expressions

- extension: robots can carry multiple containers
- extended move operator:
  move(*r*,*l*,*m*)
  - precond$_0$: adjacent(*l*,*m*), at(*r*,*l*), ¬occupied(*m*)
  - effects$_0$: at(*r*,*m*), occupied(*m*), ¬occupied(*l*), ¬at(*r*,*l*)
  - for-all *x*:
    - precond$_1$: loaded(*r*,*x*)
    - effects$_1$: at(*x*,*m*), ¬at(*x*,*l*))

Disjunctive Preconditions

- allow alternatives (disjunctions) in preconditions:
  - precond = $precond_1 \vee \ldots \vee precond_n$
  - *a* is applicable in state *s* if *s* satisfies at least one of $precond_1 \ldots precond_n$
  - effects remain unchanged
- rewrite:
  - replace operator with *n* disjunctive preconditions by *n* operators with $precond_i$ as precondition

DWR Example: Disjunctive Preconditions

- robot can move between locations if there is a road between them or the robot has all-wheel drive
- extended move operator: move(*r*,*l*,*m*)
  - precond: (road(*l*,*m*), at(*r*,*l*), ¬occupied(*m*)) ∨ (all-wheel-drive(*r*), at(*r*,*l*), ¬occupied(*m*))
  - effects: at(*r*,*m*), occupied(*m*), ¬occupied(*l*), ¬at(*r*,*l*)

Axiomatic Inference: Static Case

- axioms over rigid relations:
  - example:
    $\forall l_1,l_2$: adjacent(l_1,l_2) ↔ adjacent(l_2,l_1)
- state-specific axioms:
  - example:
    $\forall c$: container(c) ↔ at(c,loc1) holds in s_i
- approach: pre-compute

Axiomatic Inference: Dynamic Case

- axioms over flexible relations:
  - example: $\forall k,x$: ¬holding(k,x) ↔ empty(k)
  - approach:
    - divide relations into primary and secondary where secondary relations do not appear in effects
    - transform axioms into implications where primary relations must not appear in right-hand side
  - example:
    - primary: holding / secondary: empty
    - $\forall k\ \neg\exists x$: holding(k,x) → empty(k)
    - $\forall k\ \exists x$: holding(k,x) → ¬empty(k)

Extended Goals

- not part of classical planning formalisms
- some problems can be translated into equivalent classical problems, e.g.
  - states to be avoided: add corresponding preconditions to operators
  - states to be visited twice: introduce visited relation and maintain in operators
  - constraints on solution length: introduce count relation that is increased with each step

Other Extensions

- Function Symbols
  - infinite domains, undecidable in general
- Attached Procedures
  - evaluate relations using special code rather than general inference
    - efficiency may be necessary in real-world domains
    - variables must usually be bound to evaluate relations
    - semantics of such relations

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