Planning with MDPs
(Markov Decision Processes)

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Status of Classical Planning

- **Classical planning works!!**
  - *Large problems solved very fast (non-optimally)*

- **Model simple but useful**
  - *Operators not primitive; can be policies themselves*
  - *Fast closed-loop replanning able to cope with uncertainty sometimes*

- **Limitations**
  - *Does not model* **Uncertainty** *(no probabilities)*
  - *Does not deal with* **Incomplete Information** *(no sensing)*
  - *Deals with very* **Simple Cost Structure** *(no state dependent costs)*

Beyond Classical Planning: Two Strategies

1. **Develop** solver for more general models; e.g., MDPs and POMDPs

   $+$: generality
   $-$: complexity

2. **Extend** the scope of current 'classical' solvers

   $+$: efficiency
   $-$: generality

We will pursue first approach here . . .

Reminder: Basic State Models

- Characterized by:
  - finite and discrete state space S
  - an initial state $s_0 \in S$
  - a set $G \subseteq S$ of goal states
  - actions $A(s) \subseteq A$ applicable in each state $s \in S$
  - a transition function $F(a, s)$ for $s \in S$ and $a \in A(s)$
  - action costs $c(a, s) > 0$

- A **solution** is a sequence of applicable actions a_i, $i = 0, \ldots, n$, that maps the initial state s_0 into a goal state $s \in S_G$; i.e.,

$$s_{i+1} = f(a_i, s_i) \text{ and } a_i \in A(s_i) \text{ for } i = 0, \ldots, n \text{ and } s_{n+1} \in S_G$$

- **Optimal** solutions minimize total cost $\sum_{i=0}^{i=n} c(a_i, s_i)$, and can be computed by **shortest-path** or **heuristic search** algorithms . . .

Markov Decision Processes (MDPs)

MDPs are **fully observable, probabilistic** state models:

- a state space S
- a set $G \subseteq S$ of goal states
- actions $A(s) \subseteq A$ applicable in each state $s \in S$
- transition probabilities $P_a(s'|s)$ for $s \in S$ and $a \in A(s)$
- action costs $c(a, s) > 0$

– **Solutions** are **functions (policies)** mapping states into actions

– **Optimal** solutions have minimum **expected** costs

Partially Observable MDPs (POMDPs)

POMDPs are **partially observable, probabilistic** state models:

- states $s \in S$
- actions $A(s) \subseteq A$
- costs $c(a, s) > 0$
- transition probabilities $P_a(s'|s)$ for $s \in S$ and $a \in A(s)$
- initial **belief state** b_0
- final **belief states** b_F
- **sensor model** given by probabilities $P_a(o|s)$, $o \in Obs$

– **Belief states** are probability distributions over S

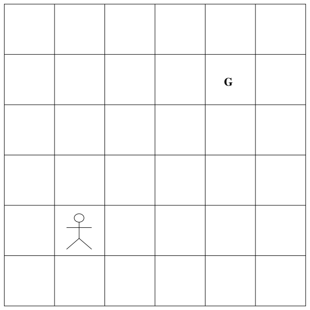
– **Solutions** are policies that map belief states into actions

– **Optimal** policies minimize **expected** cost to go from b_0 to b_F

Illustration: Navigation Problems

Consider robot that has to reach target G when

1. *initial state is known and actions are deterministic*
2. *initial state is unknown and actions are deterministic*
3. *states are fully observable and actions are stochastic*
4. *states are partially observable and actions are stochastic . . .*



– How do these problems map into the models considered?

– What is the form of the solutions?

Solving State Models by Dynamic Programming

- Solutions to **wide range** of state models can be expressed in terms of solution of **Bellman equation** over **non-terminal** states s:

$$V(s) = min_{a \in A(s)}\, Q_V(a, s)$$

where **cost-to-go** term $Q_V(a, s)$ depends on model

$$\begin{array}{ll} c(a,s) + \sum_{s' \in F(a,s)} P_a(s'|s) V(s') & \text{for MDPs} \\ c(a,s) + \max_{s' \in F(a,s)} V(s') & \text{for Max AND/OR Graphs} \\ c(a,s) + V(s'),\ s' \in F(a,s) & \text{for OR Graphs} \ldots \end{array}$$

($F(a, s)$: set of successor states; for terminal states, $V(s) = V^*(s)$ assumed)

- The **greedy policy** $\pi_V(s) = \mathrm{argmin}_{a \in A(s)}\, Q_V(a, s)$ is **optimal** when $V = V^*$ solves Bellman

- **Question:** how to get V^*?

Value Iteration (VI)

- **Value Iteration** finds V^* by successive approximations

- Starting with an arbitrary V, uses Bellman equation to update V

$$V(s) := min_{a \in A(s)} \, Q_V(a, s)$$

- If **all states updated** a sufficient number of times (and certain general conditions hold), left and right hand sides converge to $V = V^*$

- **Example:** . . .

Value Iteration: Benefits and Limitations

- VI is a very **simple** and **general** algorithm (can solve wide range of models)

- **Problem:** VI is **exhaustive**; value function $V(s)$ is a **vector of the size of the problem space**

- In particular, it does not compete with **heuristic search algorithms** such as A* or IDA* for solving OR-graphs (deterministic problems) . . .

- **Question:** can VI be 'modified' to deal with **larger state spaces** that do not fit into memory, without giving up **optimality**?

- Yes, use **Lower Bounds** and **Initial State** as in **Heuristic Search** methods . . .

Focusing Value Iteration using LBs and s_0: Find and Update

- Say that a state s is
  - **greedy** if reachable from s_0 using **greedy policy** π_V, and
  - **inconsistent** if $V(s) \neq \min_{a \in A(s)} Q_V(a, s)$
- Then starting with an **admissible** and **monotone** V, follow loop:
  - **Find** *an* **inconsistent greedy** *state* s *and* **Update** *it*
- **Find-and-Update** loop delivers greedy policy that is **optimal even if some states not updated or visited at all!**
- Recent **heuristic search algorithms for MDPs**, like RTDP, LAO*, and LDFS; all implement this loop in various forms
- We will focus here on RTDP (Barto, Bradke, Singh, 95)

Greedy Policy for For Deterministic MDP

The **Greedy policy** is a closed-loop version of greedy search

1. **Evaluate** each action a applicable in s

   $$Q(a, s) = c(a, s) + h(s_a) \text{ where } s_a \text{ is next state}$$

2. **Apply** action $\mathbf{a}$ that minimizes $Q(\mathbf{a}, s)$
3. **Observe** resulting states s'
4. **Exit** if s' is goal, else go to 1 with $s := s'$

- Greedy policy based on h can be written as $\pi_h(s) = \text{argmin}_{a \in A(s)} Q(a, s)$
- π_h is **optimal** when $h = h^*$, otherwise non-optimal and may get trapped into loops

Modifiable Greedy Policy for For Deterministic MDP (LRTA*)

Update heuristic h as you move, to make it consistent with Bellman

1. **Evaluate** each action a applicable in s

   $$Q(a, s) = c(a, s) + h(s_a) \text{ where } s_a \text{ is next state}$$

2. **Apply** action $\mathbf{a}$ that minimizes $Q(\mathbf{a}, s)$
3. **Update** $V(s)$ to $Q(\mathbf{a}, s)$
4. **Observe** resulting states s'
5. **Exit** if s' is goal, else go to 1 with $s := s'$

- Greedy policy based on h can be written as $\pi_h(s) = \text{argmin}_{a \in A(s)} Q(a, s)$
- π_h is **optimal** when $h = h^*$, otherwise non-optimal and may get trapped into loops

Real Time Dynamic Programming (RTDP)

Same as LRTA* but deals with true (probabilistic) MDP

1. **Evaluate** each action a applicable in s as

$$Q(a,s) = c(a,s) + \sum_{s' \in S} P_a(s'|s) V_i(s')$$

2. **Apply** action $\mathbf{a}$ that minimizes $Q(\mathbf{a}, s)$
3. **Update** $V(s)$ to $Q(\mathbf{a}, s)$
4. **Observe** resulting state s'
5. **Exit** if s' is goal, else go to 1 with $s := s'$

$V(s)$ initialized to $h(s)$; if $h < V^*$, RTDP eventually **optimal**

Variations on RTDP : Reinforcement Learning

Q-learning is a **model-free** version of RTDP

1. **Apply** action $\mathbf{a}$ that minimizes $Q(\mathbf{a}, s)$ with probability $1-\epsilon$, with probability ϵ, choose $\mathbf{a}$ randomly
2. **Observe** resulting state s'
3. **Update** $Q(\mathbf{a}, s)$ to

$$(1-\alpha)Q(\mathbf{a}, s) + \alpha[c(\mathbf{a}, s) + \max_a Q(a, s')]$$

4. **Exit** if s' is goal, else with $s := s'$ go to 1

Q-learning learns asymptotically to solve MDPs optimally (Watkins 89)

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