Neuron receives **inputs** and **combines** these in the cell body. If the input reaches a **threshold**, then the neuron may **fire** (produce an output). Some inputs are **excitatory**, while others are **inhibitory**.
In biological neural networks, connections are **synapses**.

- **Input connection** is conduit through which a member of a network receives information (INPUT).
- **Output connection** is a conduit through which a member of a network sends information (OUTPUT).
Connectionism is the name for a computer modeling approach based on how information processing occurs in neural networks (connectionist networks are called artificial neural networks).
Perceptron was developed by Frank Rosenblatt in 1957 and can be considered as the simplest artificial neural network.
Perceptron: An Artificial Neuron

Perceptron was developed by Frank Rosenblatt in 1957 and can be considered as the simplest artificial neural network.

\[ u(x) = \sum_{i=1}^{n} w_i x_i \]

Input function:

Activation function: threshold

\[ y = f(u(x)) = \begin{cases} 1, & \text{if } u(x) > \theta \\ 0, & \text{otherwise} \end{cases} \]

Activation state: 0 or 1 (-1 or 1)
Perceptron was developed by Frank Rosenblatt in 1957 and can be considered as the simplest artificial neural network.

\[ u(x) = \sum_{i=1}^{n} w_i x_i \]

Activation function: threshold

\[ y = f(u(x)) = \begin{cases} 1, & \text{if } u(x) > \theta \\ 0, & \text{otherwise} \end{cases} \]
Perceptron was developed by Frank Rosenblatt in 1957 and can be considered as the simplest artificial neural network.

Input function:
\[ u(x) = \sum_{i=1}^{n} w_i x_i \]

Activation function: threshold
\[ y = f(u(x)) = \begin{cases} 1, & \text{if } u(x) > \theta \\ 0, & \text{otherwise} \end{cases} \]

Activation state: 0 or 1 (-1 or 1)
Perceptron was developed by Frank Rosenblatt in 1957 and can be considered as the simplest artificial neural network.

- Inputs are in the range $[0, 1]$, where 0 is “off” and 1 is “on”.
- Weights can be any real number (positive or negative).
Perceptrons for Logic

Perceptron for AND

\[ f \text{ if } \sum \geq \theta \text{ then } 1 \text{ else } 0 \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ AND } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for AND

\[ f \text{ if } \sum \geq \theta \text{ then } 1 \text{ else } 0 \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ AND } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for AND

\[ f \text{ if } \sum \geq \theta \text{ then } 1 \text{ else } 0 \]

\[ 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5 \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ AND } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

**Perceptron for AND**

\[ f \text{ if } \sum_{i} x_i \geq \theta \text{ then } 1 \text{ else } 0 \]

\[ 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5 \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ AND } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for AND

\[ f \text{ if } \sum x_i \geq \theta \text{ then } 1 \text{ else } 0 \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>( x_1 \text{ AND } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for AND

1

0.5

0.5

1 $f$ if $\sum \geq \theta$ then 1 else 0

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_1 \text{ AND } x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for AND

\[ f \text{ if } \sum \geq \theta \text{ then 1 else 0} \]

\[ 1 \cdot 0.5 + 1 \cdot 0.5 = 1 \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ AND } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for AND

\[ f(x_1, x_2) = \begin{cases} 1 & \text{if } \sum x_i \geq \theta \text{ then } 1 \text{ else } 0 \\ 1 \cdot 0.5 + 1 \cdot 0.5 = 1 \end{cases} \]

\[
\begin{array}{c|c|c}
 x_1 & x_2 & x_1 \text{ AND } x_2 \\
 0 & 0 & 0 \\
 0 & 1 & 0 \\
 1 & 0 & 0 \\
 1 & 1 & 1 \\
\end{array}
\]
Perceptrons for Logic

Perceptron for OR

\[ f(x) = \begin{cases} 1 & \text{if } \sum x_i \geq \theta \\ 0 & \text{otherwise} \end{cases} \]

<table>
<thead>
<tr>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_1 \ OR \ x_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for OR

0.5

0.5

0.5

f if \( \sum \geq \theta \) then 1 else 0

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 ) OR ( x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for OR

\[ f \text{ if } \sum \geq \theta \text{ then 1 else 0} \]

\[ 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5 \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ OR } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for OR

\[ f \text{ if } \sum x_i \geq \theta \text{ then } 1 \text{ else } 0 \]

\[ 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5 \]

<table>
<thead>
<tr>
<th>x_1</th>
<th>x_2</th>
<th>x_1 \text{ OR } x_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Perceptrons for Logic

Perceptron for XOR

\[ f \text{ if } \sum \geq \theta \text{ then } 1 \text{ else } 0 \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ XOR } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

XOR is an **exclusive OR** because it only returns a **true** value of 1 if the two values are exclusive, i.e., they are both different.
XOR is an **exclusive OR** because it only returns a **true** value of 1 if the two values are exclusive, i.e., they are both different.
Perceptrons for Logic

Perceptron for XOR

0.5

f if \( \sum \geq \theta \) then 1 else 0

0 · 0.5 + 1 · 0.5 = 0.5

XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.
XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.
Perceptrons for Logic

Perceptron for XOR

\[ f \text{ if } \sum \geq \theta \text{ then } 1 \text{ else } 0 \]

XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.
XOR is an **exclusive OR** because it only returns a **true** value of 1 if the two values are exclusive, i.e., they are both different.
Perceptrons for Logic

Perceptron for XOR

\[ f(\sum \geq \theta) = 1 \text{ if } \sum \geq \theta \text{ else } 0 \]

\[ 0 \cdot 0.5 + 0 \cdot 0.5 = 0 \]

XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \text{ XOR } x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.
Perceptrons for Logic

Perceptron for XOR

1

0.5

0.5

0.5

1 \cdot 0.5 + 1 \cdot 0.5 = 1

XOR is an \textit{exclusive OR} because it only returns a \textit{true} value of 1 if the two values are exclusive, i.e., they are both different.
XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.
Perceptrons as Classifiers

\[ x = \sum_{i=0}^{n} w_i x_i \]

Diagram:
- Input nodes: \( x_1, x_2, \ldots, x_n \)
- Weights: \( w_0, w_1, \ldots, w_n \)
- Output: \( y \)
- Function: \( f(x) = 1 \) for \( x > 0 \), \( f(x) = 0 \) otherwise

-1

1
Perceptrons are *linear* classifiers, i.e., they can only separate points with a *hyperplane* (a straight line).
The XOR problem again
What is the Perceptron Really Seeing?

Sequence of exemplars presented to the Perceptron:

<table>
<thead>
<tr>
<th>N</th>
<th>input x</th>
<th>target t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,1,0,0)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(1,0,0,0)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>(0,1,1,1)</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>(1,0,1,0)</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>(1,1,1,1)</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>(0,1,0,0)</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- The above Perceptron has 4 inputs (binary) ≈ feature vector representing each exemplar.
- The Perceptron see 6 exemplars or training items
- We **know** what the right answer is ≈ target
What is the Perceptron Really Seeing?

Sequence of exemplars presented to the Perceptron:

<table>
<thead>
<tr>
<th>N</th>
<th>input x</th>
<th>target t</th>
<th>output o</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,1,0,0)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(1,0,0,0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>(0,1,1,1)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>(1,0,1,0)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>(1,1,1,1)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>(0,1,0,0)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- The above Perceptron has 4 inputs (binary) \(\approx\) feature vector representing each exemplar.
- The Perceptron see 6 exemplars or training items
- We **know** what the right answer is \(\approx\) target
What is the Perceptron Really Seeing?

Sequence of exemplars presented to the Perceptron:

<table>
<thead>
<tr>
<th>N</th>
<th>input x</th>
<th>target t</th>
<th>output o</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,1,0,0)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(1,0,0,0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>(0,1,1,1)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>(1,0,1,0)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>(1,1,1,1)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>(0,1,0,0)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- The above Perceptron has 4 inputs (binary) \(\approx\) feature vector representing each exemplar.
- The Perceptron see 6 exemplars or training items
- We know what the right answer is \(\approx\) target
- What would happen if we used random weights/threshold?
**Q$_1$:** But... choosing weights and threshold $\theta$ for the perceptron is not easy! How to learn the weights and threshold from examples?

**A$_1$:** We can use a **learning algorithm** that adjusts the weights and threshold based on examples.

http://www.youtube.com/watch?v=vGwemZhPlsA&feature=youtu.be
Learning: A trick to learn $\theta$

$$\sum_{i=1}^{n} w_i x_i > \theta$$
Learning: A trick to learn $\theta$

\[
\sum_{i=1}^{n} w_i x_i > \theta
\]

\[
\sum_{i=1}^{n} w_i x_i - \theta > 0
\]
Learning: A trick to learn $\theta$

\[ \sum_{i=1}^{n} w_i x_i > \theta \]

\[ \sum_{i=1}^{n} w_i x_i - \theta > 0 \]

\[ w_1 x_1 + w_2 x_2 + \ldots + w_n x_n - \theta > 0 \]
Learning: A trick to learn $\theta$

$$\sum_{i=1}^{n} w_i x_i > \theta$$

$$\sum_{i=1}^{n} w_i x_i - \theta > 0$$

$$w_1 x_1 + w_2 x_2 + \ldots + w_n x_n - \theta > 0$$

$$w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + \theta(-1) > 0$$
Learning: A trick to learn $\theta$

$\sum_{i=1}^{n} w_i x_i > \theta$

$\sum_{i=1}^{n} w_i x_i - \theta > 0$

$w_1 x_1 + w_2 x_2 + \ldots + w_n x_n - \theta > 0$

$w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + \theta(-1) > 0$
We can consider $\theta$ as a weight to be learnt!

The input is fixed as -1. The activation function is then:

$$y = f(u(x)) = \begin{cases} 1, & \text{if } u(x) > 0 \\ 0, & \text{otherwise} \end{cases}$$
Learning Rule

Learning happens by adjusting weights. The threshold can be considered as a weight.

\[
\Delta w_i = \eta(t - o)x_i
\]

- \( \eta, \ 0 < \eta \leq 1 \) is a constant called learning rate.
- \( t \) is the target output of the current example.
- \( o \) is the output obtained by the Perceptron.
Learning Rule

Perceptron’s Learning Rule

\[ w_i \leftarrow w_i + \Delta w_i \]

\[ \Delta w_i = \eta(t - o)x_i \]

- \( o = 1 \) and \( t = 1 \)
- \( o = 0 \) and \( t = 1 \)

- Learning rate \( \eta \) is positive; controls how big changes \( \Delta w_i \) are.
- If \( x_i > 0 \), \( \Delta w_i > 0 \). Then \( w_i \) increases in an attempt to make \( w_i x_i \) become larger than \( \theta \).
- If \( x_i < 0 \), \( \Delta w_i < 0 \). Then \( w_i \) reduces.
### Learning Rule

#### Perceptron’s Learning Rule

\[ w_i \leftarrow w_i + \Delta w_i \]

\[ \Delta w_i = \eta(t - o)x_i \]

<table>
<thead>
<tr>
<th>Condition</th>
<th>( \Delta w_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( o = 1 ) and ( t = 1 )</td>
<td>( \Delta w_i = \eta(t - o)x_i = \eta(1 - 1)x_i = 0 )</td>
</tr>
<tr>
<td>( o = 0 ) and ( t = 1 )</td>
<td>( \Delta w_i = \eta(t - o)x_i = \eta(1 - 0)x_i = \eta x_i )</td>
</tr>
</tbody>
</table>

- Learning rate \( \eta \) is positive; controls how big changes \( \Delta w_i \) are.
- If \( x_i > 0 \), \( \Delta w_i > 0 \). Then \( w_i \) increases in an attempt to make \( w_i x_i \) become larger than \( \theta \).
- If \( x_i < 0 \), \( \Delta w_i < 0 \). Then \( w_i \) reduces.
Learning Algorithm

1: Initialize all weights randomly.
2: repeat
3: for each training example do
4: Apply the learning rule.
5: end for
6: until the error is acceptable or a certain number of iterations is reached

This algorithm is guaranteed to find a solution with error zero in a limited number of iterations as long as the examples are linearly separable.
MLPs are **feed-forward** neural networks, organized in layers.

One **input** layer, one or more **hidden** layers, one **output** layer.

Each node in a layer connected to all other nodes in next layer.

Each connection has a weight (can be zero).
Activation Functions

Step function

Outputs 0 or 1.

Sigmoid function

Outputs a real value between 0 and 1.
Sigmoids

1/(1+exp(-x))

1/(1+exp(5x))

1/(1+exp(0.1x))

1/(1+exp(1.5x))

1/(1+exp(-0.1x))

1/(1+exp(-5x))
1 Present the pattern at the input layer.
1. Present the pattern at the input layer.
2. Calculate activation of input neurons
1. Present the pattern at the input layer.
2. Calculate activation of input neurons
1. Present the pattern at the input layer.
2. Calculate activation of input neurons
Forward Pass

1. Present the pattern at the input layer.
2. Calculate activation of input neurons
1 Present the pattern at the input layer.
2 Calculate activation of input neurons
3 Propagate forward activations step by step.
1. Present the pattern at the input layer.
2. Calculate activation of input neurons
4. Read the network output from both output neurons.
General Idea

1. Send the MLP an input pattern, $x$, from the training set.
2. Get the output from the MLP, $y$.
3. Compare $y$ with the “right answer”, or target $t$, to get the error quantity.
4. Use the error quantity to modify the weights, so next time $y$ will be closer to $t$.
5. Repeat with another $x$ from the training set.

When updating weights after seeing $x$, the network doesn’t just change the way it deals with $x$, but other inputs too . . .

Inputs it has not seen yet!

Generalization is the ability to deal accurately with unseen inputs.
Recall: Perceptron Learning Rule

Minimize the difference between the actual and desired outputs:

\[ w_i \leftarrow w_i + \eta(t - o)x_i \]

Error Function: Mean Squared Error (MSE)

An error function represents such a difference over a set of inputs:

\[ E(\vec{w}) = \frac{1}{2N} \sum_{p=1}^{N} (t^p - o^p)^2 \]

- \( N \) is the number of patterns
- \( t^p \) is the target output for pattern \( p \)
- \( o^p \) is the output obtained for pattern \( p \)
- the 2 makes little difference, but makes life easier later on!
Interpret $E$ just as a mathematical function depending on $\vec{w}$ and forget about its semantics, then we are faced with a problem of mathematical optimization.

$$\minimize_{\vec{u}} f(\vec{u})$$

We consider only continuous and differentiable functions.
Gradient descent can be used for minimizing functions.

The derivative is a measure of the rate of change of a function, as its input changes;

For function $y = f(x)$, the derivative $\frac{dy}{dx}$ indicates how much $y$ changes in response to changes in $x$.

If $x$ and $y$ are real numbers, and if the graph of $y$ is plotted against $x$, the derivative measures the slope or gradient of the line at each point, i.e., it describes the steepness or incline.
So, we know how to use derivatives to adjust one input value. But we have several weights to adjust! We need to use partial derivatives.

A partial derivative of a function of several variables is its derivative with respect to one of those variables, with the others held constant.

**Example**

If \( y = f(x_1, x_2) \), then we can have \( \frac{\partial y}{\partial x_1} \) and \( \frac{\partial y}{\partial x_2} \).

In our learning rule case, if we can work out the partial derivatives, we can use this rule to update the weights:

\[
w'_{ij} = w_{ij} + \Delta w_{ij}
\]

where \( \Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} \).
Learning Rate
Small $\eta$ leads to convergence.
Very small $\eta$, convergence may take very long.
Case of medium size $\eta$, also converges.
Very large $\eta$: divergence.
Pure gradient descent is a nice theoretical framework but of limited power in practice.

Finding the right $\eta$ is annoying. Approaching the minimum is time consuming.

Heuristics to overcome problems of gradient descent:
- gradient descent with momentum
- individual learning rates for each dimension
- adaptive learning rates
- decoupling step length from partial derivates
We learnt what a multilayer perceptron is.
We know a learning rule for updating weights in order to minimise the error:

\[ w'_{ij} = w_{ij} + \Delta w_{ij} \]

where \( \Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} \)

\( \Delta w_{ij} \) tells us in which direction and how much we should change each weight to roll down the slope (descend the gradient) of the error function \( E \).

So, how do we calculate \( \frac{\partial E}{\partial w_{ij}} \)?
The mean squared error function $E$, which we want to minimize:

$$E(\vec{w}) = \frac{1}{2N} \sum_{p=1}^{N} (t^p - o^p)^2$$
If we use a sigmoid activation function $f$, then the output of neuron $i$ for pattern $p$ is:

$$o_i^p = f(u_i) = \frac{1}{1 + e^{au_i}}$$

where $a$ is a pre-defined constant and $u_i$ is the result of the input function in neuron $i$:

$$u_i = \sum_j w_{ij}x_{ij}$$
For the $p$th pattern and the $i$th neuron, we use gradient descent on the error function:

$$\Delta w_{ij} = -\eta \frac{\partial E_p}{\partial w_{ij}} = \eta (t_i^p - o_i^p) f'(u_i)x_{ij}$$

where $f'(u_i) = \frac{df}{du_i}$ is the derivative of $f$ with respect to $u_i$. If $f$ is the sigmoid function, $f'(u_i) = af(u_i)(1 - f(u_i))$. 

$$\sum_j w_{ij}$$
Using Gradient Descent to Minimize the Error

We can update weights after processing each pattern, using rule:

$$\Delta w_{ij} = \eta (t_i^p - o_i^p) f'(u_i) x_{ij}$$

$$\Delta w_{ij} = \eta \delta_i^p x_{ij}$$

- This is known as the **generalized delta rule**.
- We need to use the derivative of the activation function $f$.
  So, $f$ must be differentiable!
- Sigmoid has a derivative which is easy to calculate.
Using Gradient Descent to Minimize the Error

We can update weights after processing each pattern, using rule:

\[ \Delta w_{ij} = \eta \left( t_i^p - o_i^p \right) f'(u_i) x_{ij} \]

\[ \Delta w_{ij} = \eta \delta_i^p x_{ij} \]

- This is known as the **generalized delta rule**.
- We need to use the derivative of the activation function \( f \).
  So, \( f \) must be differentiable!
- Sigmoid has a derivative which is easy to calculate.
We can update output neurons using the generalize delta rule:

\[ \Delta w_{ij} = \eta \, \delta^p_i \, x_{ij} \]

\[ \delta^p_i = (t^p_i - o^p_i)f'(u_i) \]

This \( \delta^p_i \) is only good for the output neurons, it relies on target outputs. But we don’t have target output for the hidden nodes!

\[ \Delta w_{ki} = \eta \, \delta^p_k \, x_{ik} \]

\[ \delta^p_k = \sum_{j \in I_k} \delta^p_j \, w_{kj} \]

This rule propagates error back from output nodes to hidden nodes. If effect, it blames hidden nodes according to how much influence they had. So, now we have rules for updating both output and hidden neurons!
1. Present the pattern at the input layer.
1. Present the pattern at the input layer.
2. Propagate forward activations
1 Present the pattern at the input layer.
2 Propagate forward activations step
1. Present the pattern at the input layer.
2. Propagate forward activations step by step.
1 Present the pattern at the input layer.
2 Propagate forward activations step by step.
1. Present the pattern at the input layer.
2. Propagate forward activations step by step.
3. Calculate error from both output neurons.
1. Present the pattern at the input layer.
2. Propagate forward activations step by step.
3. Calculate error from both output neurons.
4. Propagate backward error.
1. Present the pattern at the input layer.
2. Propagate forward activations step by step.
3. Calculate error from both output neurons.
4. Propagate backward error.
1. Present the pattern at the input layer.
2. Propagate forward activations step by step.
3. Calculate error from both output neurons.
4. Propagate backward error.
1. Present the pattern at the input layer.
2. Propagate forward activations step by step.
3. Calculate error from both output neurons.
4. Propagate backward error.
5. Calculate \( \frac{\partial E}{\partial w_{ij}} \); repeat for all patterns and sum up.
1: Initialize all weights to small random values.
2: repeat
3: for each training example do
4: Forward propagate the input features of the example to determine the MLP’s outputs.
5: Back propagate error to generate $\Delta w_{ij}$ for all weights $w_{ij}$.
6: Update the weights using $\Delta w_{ij}$.
7: end for
8: until stopping criteria reached.
We learnt what a multilayer perceptron is.
We have some intuition about using gradient descent on an error function.
We know a learning rule for updating weights in order to minimize the error:
$$\Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}}$$
If we use the squared error, we get the generalized delta rule:
$$\Delta w_{ij} = \eta \delta^p_i x_{ij}.$$  
We know how to calculate $\delta^p_i$ for output and hidden layers.
We can use this rule to learn an MLP’s weights using the backpropagation algorithm.