Statistical models in neuroscience

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Data analysis with the pairwise maximum entropy model
Multi-information in an interacting system

Idea: hierarchy of entropies

\[ S_1 \geq S_2 \geq S_3 \geq \ldots \geq S_N = S(\{\sigma\}) \]

Joint entropy:

\[ S(\{\sigma\}) = -\sum_{\{\sigma_i\}} p(\sigma_i) \log p(\sigma_i) \leq \sum_{j=1}^{N} S(\sigma_j) \]

Multi-information: Entropy difference between independent model and full system, or \( D_{KL} \) between joint distribution and independent model produced from marginals \( \prod_j p(\sigma_i) \):

\[ I(\sigma) = \sum_j S(\sigma_j) - S(\{\sigma\}) = \sum_{\sigma_i} p(\sigma_i) \log \frac{p(\sigma_i)}{\prod_j p(\sigma_j)} \]
Decomposition of multi-information

\[ I(\{\sigma_i\}) = \sum_{k=2}^{N} I_C^{(k)}(\{\sigma_i\}) \]

With connected information:

\[ I_C^{(k)}(\{\sigma_i\}) = S \left[ \tilde{p}^{(k-1)}(\{\sigma_i\}) \right] - S \left[ \tilde{p}^{(k)}(\{\sigma_i\}) \right] \geq 0 \]

Where \( \tilde{p}^{(k)}(\{\sigma_i\}) \) is the ME distribution consistent with \( k^{th} \)-order marginals.

This allows to measure the contributions of all orders to the total entropy difference between independent model and data.

See Schneidman et al. (2003) for examples.
Multi-information in retinal recordings from Schneidman et al. (2006)

But note: relative measures such as \( \frac{I_2}{I_N} \) improve with very small time bins, low firing rates or small groups (Roudi et al., 2009). It is therefore not always possible to extrapolate from small models.
Inferring functional connectedness

Schneidman et al. (2006)
Error correction

- Spiking can be unreliable.
- Suppose 1 neuron, 2 stimuli $S = \{1, 2\}$, $p(S = 1) = 0.5$
- Correct response: $p(\sigma_1 = 1|S = 1) = p_{S1}$
- Incorrect: $p(\sigma = -1|S = 1) = 1 - p_{S1}$
- Error probability for 3 identical neurons:

$$ (1 - p_{S1})^3 + 3(1 - p_{S1})^2 p_{S1} < 1 - p_{S1} $$

- Simple majority rule yields correct answers for large $N$.
- But potentially inefficient.
Error correction

Quantifies the entropy reduction through correlations: \( I_N = S_1 - S_N \)

When \( I_N \to S_1 \), correlations dominate the activity.

Schneidman et al. (2006) find: \( I_N \to S_1 \) for \( N \approx 200 \)
Error correction

Information $N$ cells provide about the $(N+1)$th cell. But: extrapolated from $N = 15 \pm$ natural (correlated) stimuli
Schneidman et al. (2006)
Model extensions: Markov model

One time step constraint:
\[ < \sigma_i(t)\sigma_j(t + 1) >_t = \frac{1}{T} \sum_{t=1}^{T} \sigma_i(t)\sigma_j(t + 1) = c_{ij}^1 \]

\[ P(\{\sigma\}^t, \{\sigma\}^{t+1}) = \frac{\exp\left(\sum_{j=1}^{N} h_j \sigma^t_j + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k \right)}{\sum_{i=1}^{M} \exp\left(\sum_{j=1}^{N} h_j \sigma^t_j + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k + \frac{1}{2} \sum_{j\neq k}^{N} J^t_{jk} \sigma^t_j \sigma^t_k \right)} \]

Higher orders in time are possible, then we have a n-step Markov model.

This is a special case of a GLM.

Marre et al. (2009)
Model extensions: Markov model

Marre et al. (2009)
Model extensions: K-pairwise model

Tkačik et al. (2014)
Model extensions: K-pairwise model

Add constraint on population firing rate:

$P_N(K)$: probability that $K$ of $N$ neurons fire simultaneously

$$P_N(K) = \left< \delta \left( \sum_{i=1}^{N} \sigma_i , 2K - N \right) \right>$$

$$\implies$$

$$P(\{\sigma\}) = \frac{1}{Z} \exp \left( E_{pair} + \sum_{K=1}^{N} \lambda_K \left( \sum_{j=1}^{N} \sigma_j \right)^K \right)$$

Tkačik et al. (2014)
Model extensions: K-pairwise model

Good match of higher order structure

Tkačik et al. (2014)
Energy landscapes

Identified metastable states.

Tkačik et al. (2014)
Metastable states

Identified metastable states.
Tkačik et al. (2014)
Reproducible sequences of metastable states

Tkačik et al. (2014)


