Neural Information Processing: 2009-2010
Assignment 1

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1st March 2010

Remember that plagiarism is a university offence. Please read the policy at http://www.inf.ed.ac.uk/admin/ITO/DivisionalGuidelinesPlagiarism.html .

Practical information

You should produce a digital document for your assignment answers (e.g. with latex) and submit this electronically using the `submit` command on a DICE machine. The format is e.g.

```
submit
msc nip 1 nipasst1.pdf
```

You can check the status of your submissions with the `show_submissions` command. NOTE: postscript or pdf formats are acceptable, other formats are not. Make sure that the file you submit prints ok on the DICE system, in particular when you produced it on a non-Unix machine.

Is not necessary to submit the code.

Late submissions:

Late submissions will receive a zero mark. Only evidence for illness or other serious reasons can prevent this at the discretion of the instructor. See

`http://www.inf.ed.ac.uk/teaching/years/msc/courseguide09.html\#exam`

The handout "Introduction to MATLAB" available from the PMR webpage may be helpful if you are not very familiar with MATLAB. Recall that the current figure window can be saved as an encapsulated postscript file `myplot.eps` using the command `print -deps2 myplot.eps`.

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Population codes and estimators

In this assignment we re-analyse the population coding model of the cricket wind receptors. This model is described briefly in the lecture notes, the Dayan and Abbott book and in [Salinas and Abbott, 1994]. In the model there are 4 neurons.

At each trial the firing rates, r_i, for neuron $i = 1 \ldots 4$ are modelled as

$$r_i = f_i(\theta) + \sigma\eta_i$$

where η_i are independent Gaussian random variables, all with standard deviation σ, which we take to be $\sigma = 0.1$. The tuning curves are given by

$$f_i(\theta) = [\cos(\theta - \phi_i)]_+$$

where the preferred angle of neuron i is given by $\phi_i = \frac{\pi}{4} + \frac{2\pi i}{N}$, and with notation $[x]_+ = \max(x, 0)$.

Question 1: Implement the network and implement a population vector readout. Plot the trial-to-trial std.dev. in the estimate as a function of the encoded angle.

Question 2: Estimate the error in the population vector based estimate in the limit of small noise. To do this assume that only 2 neurons are active, so that the population vector equals $\mathbf{v} = (\cos(\theta) + \sigma\eta_1, \sin(\theta) + \sigma\eta_2)$, where η_1 and η_2 are independent Gaussian variables[1]. From the population vector, extract the angle estimate. Using a Taylor expansion, calculate the variance in the estimated angle to first order of σ^2.

Question 3: Implement a Maximum Likelihood estimator of the encoded angle. Do this as far analytically as possible.
Describe what you did and plot the std.dev. in the estimate as a function of the encoded angle.
Can the likelihood have local maxima?

Question 4: Calculate the lower bound on the std.dev. using the Cramer-Rao bound. Instead of calculating the probability integral numerically, again do this as far analytically as possible.
Compare to the result of question 3.

Question 5: A somewhat more realistic model is to assume that

[1] We rotate the coordinate system, depending on which neurons were active, so that the active neurons align with the x- and y-axes. This simplifies matters.

$$r_i = [\cos(\theta - \phi_i) + \sigma\eta]_+$$

so that the firing rate is never negative, even when noise is present. Write down $P(\mathbf{r}|\theta)$ in this case.

Practical details

It is probably best to use MatLab for this assignment, although other languages are fine as well.

You will find that you need a large number of trials (>1000) for each encoded angle that you test. Therefore, efficient coding of some parts is worthwhile. You can profile you Matlab code with the PROFILE ON command (see help) to see where the bottlenecks are.

To minimize functions, I have seen differences in performance between FMINSEARCH than FMINBND. The latter finds less accurate minima, it seems. You have been warned...

Finally, because the noise model used here differs from [Salinas and Abbott, 1994], the results will not look identical to theirs.

References

[Salinas and Abbott, 1994] Salinas, E. and Abbott, L. F. (1994). Vector reconstruction from firing rates. *J. of Comput. Neurosc.*, 1:89–107.