Assignment 2: Variants of the BCM-rule

Neural Computation 2017-2018. Mark van Rossum

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Practical info

Organize your answers according to the questions; don't merge them. Plots should include axis labels and units (either on the plot, or mentioned in the text), see the link on the course website. Some answers might require units as well.

You will find that some questions are quite open-ended. In order to receive full marks for those you will need to do more than running a simulation and making a plot. Instead, you should substantiate your explanations and claims, for instance by doing additional simulations or mathematical analysis. However, core-dumping (just writing down all you can think of) is discouraged, and incorrect claims can reduce marks. It should not be necessary to consult scientific literature, but if you do use additional literature, cite it. There will be a to-be-determined normalization factor between the number of points scored and the resulting percentage mark.

Copying results is absolutely not allowed and can lead to severe punishment. It's OK to ask for help from your friends. However, this help must not extend to copying code, results, or written text that your friend has written, or that you and your friend have written together. I assess you on the basis of what you are able to do by yourself. It's OK to help a friend. However, this help must not extend to providing your friend with code or written text. If you are found to have done so, a penalty will be assessed against you as well.

Deadline will be announced via email and the website.

Submit using the submit system: `http://computing.help.inf.ed.ac.uk/submit`

Model and setup

In this assignment we consider the BCM plasticity rule for a single neuron receiving just two inputs.

Given the two-dimensional input vector $\boldsymbol{x}$, the neuron's activity is $y = \boldsymbol{w}.\boldsymbol{x}$, where $\boldsymbol{w}$ are the synaptic weights, written as a vector. The activities as well as weights are allowed to be negative.

The weights are plastic according to the BCM -rule

$$\frac{d\boldsymbol{w}}{dt} = \eta \boldsymbol{x} y (y - \theta)$$

The threshold θ tracks the recent value of the activity y^2, and obeys

$$\tau_\theta \frac{d\theta}{dt} = -\theta + y^2$$

The input alternates between two values $\boldsymbol{x}^a = (\cos\phi_a, \sin\phi_a)$ and $\boldsymbol{x}^b = (\cos\phi_b, \sin\phi_b)$, choose $\phi_a = 0$, $\phi_b = \pi/3$.

Simulations of the system contain the following steps: select the input, calculate the neural activity, update the weights, and update the threshold.

Parameters: weight update $\eta = 0.01$, $\tau_\theta = 10$ (timesteps), at least 1000 total simulation steps.

Questions

Question 1 Simulate the system and plot the resulting activity, threshold and weights versus time. Comment on you findings.

Question 2 Show mathematically, that unless $\boldsymbol{x}^a \propto \boldsymbol{x}^b$, in the steady state the weights are such that $\boldsymbol{x}^a.\boldsymbol{w}$ equals zero or θ, and $\boldsymbol{x}^b.\boldsymbol{w}$ equals zero or θ (you can assume a fixed θ for this). Compare to your simulation results.

Question 3 Explore what happens in the simulations if you make τ_θ larger and comment on your findings.

Question 4 As variant of the rule simulate the following rule $\frac{dw_i}{dt} = \eta x_i y (y - \theta |w_i|)$. What are the steady state weights under that rule?