Notes to answers Assignment 2

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Question 1 (5 points) Numerically calculate and plot the total weight change $\Delta w = \Delta w_+ + \Delta w_-$ as a function of Δt (=$-20 \ldots +20$ms) for a couple of pairing frequencies f_{pair} in the range $1 \ldots 100$Hz. Take 20 pre-post spike pairs. Comment on your findings and the various contributions to Δw. Note, that this calculation does not require any neural model, but just the implementation of the spike trains, the traces, and the weight change.

- The plot Δw vs Δt is a (almost) periodic plot, which looks like a sawtooth. The periodicity comes from the fact that when when you shift the spike train with $\Delta t = 1/f$, you have virtually the same spike train (minus one pairing event).
- Note that at low frequencies only LTD remains, while at high pairing frequencies, LTP dominates. In extreme cases there is nowhere LTD.

Question 2 (5 points) Calculate analytically the general expression for the weight change per presynaptic spike for the setup of the previous question. Ignore the first pre-post pair, as for that one $y(t) = 0$. Compare your results to Question 1.

- Of course all traces are decaying exponential functions. The periodicity can cause confusion. It is easiest to take $0 \leq \Delta t \leq 1/f$, and make the function periodic afterwards. In that the pre-post time is Δt and the post-pre time is $1/f - \Delta t$.
- LTD-term. Here you need $y(t - \epsilon)X(t)$. (The ϵ can be safely ignored). The trace y at time of the pre-spike is $\exp(-(1/f - \Delta t)/\tau_y)$.
- For the LTP we need $x(t)y(t - \epsilon)Y(t)$. The second term at the time of the post-spike is $y = \exp(-1/(f\tau_y))$.
- Of course, this should match the simulations well..
- A common mistake was to assume that there was only LTP when $\Delta t > 0$ (and only LTD when $\Delta t < 0$). This is not tru due to the periodic nature of the problem; for any Δt both LTD and LTP occur.

Question 3 (5 points) Repeat Question 1 for the case that both spike trains are Poisson trains with rates ρ_x and ρ_y respectively. Plot the change over a one second interval for a couple of settings of ρ_x and ρ_y. Comment on your findings.

Question 4 (5 points) Calculate the average weight change per pre-synaptic spike in case both spike trains are Poisson trains.

- We look at the (more difficult) LTP term $x(t)y(t-\epsilon)Y(t)$. We need to average over all possible configuration of the spike times. $\langle \Delta w_+ \rangle = \int_o^\infty P(t_y)P(\Delta t)\Delta w(t_y,\Delta t) d\Delta t\, dt_y$, where $P(t_y) = \rho_y \exp(-t_y \rho_y)$ is the time since the last post-spike. As this an Poisson process, the interval distribution is exponential. Similarly, $P(\Delta t) = \rho_x \exp(-\Delta t \rho_x)$ is the time since the last pre-spike (being a Poisson process it does not matter what we take as measurement time; remember the derivation). Also, $\Delta w(t_y, \Delta t) = c_+ \exp(-t_y/\tau_y)\exp(-\Delta t/\tau_x)$.
  This gives $\langle \Delta w_+ \rangle = c_+ \frac{\rho_x \rho_y}{(\rho_x+1/\tau_x)(\rho_y+1/\tau_y)}$ (per post spike). For the LTD-term: $\langle \Delta w_- \rangle = c_- \frac{\rho_x}{(\rho_x+1/\tau_x)}$ (per pre spike).

- A common mistake was to calculate the average values of Δt and t_y first, and then plug them into the equation for $\Delta w(t_y, \Delta t)$. While this type of error is common it is never allowed!

Part 2: Post vs pre expression of plasticity

Question 5 (5 points) Create an integrate and fire neuron that receives a 1 second long periodic train of pulses at 50Hz through a synapse with release modelled as above. The synaptic input is modelled as a current, decaying with a time-constant τ_{syn} between inputs and increasing an amount $r(t)UA$ for every input pulse. Parameters are as in the practical, further use a 5ms synaptic time-constant and $\tau_D = 200$ms. Take A about 5000, so that the neuron has a decent response. Compare the effect on the firing rate of the neuron when A is doubled vs a doubling of U. Make sure to study the onset transient in the response as well.

- With a doubling of A the steady state response is roughly doubled, but doubling of U has a smaller effect. The transients (say, within the first 100ms after stimulus onset) are however comparable.

- Common error 1: the release probability should be initialized to 1 (a 'well-rested' synapse), not 0.

- Common error 2: some found an 'inverted U' curve when plotting frequency vs U. This is wrong, frequency should always increase with U, albeit in a saturating manner. Probably caused by updating r before updating the synaptic current.

Question 6 (5 points) Assume a very long train of pre-synaptic spikes. What is the steady state synaptic strength during this prolonged stimulation? Does this explain the result from Question 5?

- The idea is that in state steady, the synapse recovers between events as much as it loses per pre-synaptic event. Right after a spike $r_0 = 1 - c$, while right before the (next) spike it is $r_1 = 1 - c\exp(-T/\tau_D)$. At the same time $r_1 - r_1 U = r_0$. This gives $r_1 = \frac{1-e}{1-e+Ue}$, where $e = \exp(-T/\tau_d) = \exp(-1/(f\tau_d))$.

- The weight is proportional to Ur_1. This is a increasing, saturating function of U. It explains that increasing U increases the steady-state strength of the synapse, but less than changes in A.