Assignment 2: Rate based learning rules

Neural Computation 2005-2006. Mark van Rossum

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Practical info

Report your findings. Plots should include axes labels and units (either on the plot, or mentioned in the text), see my web page link on reports! Particularly well-researched answers can receive additional points. There will be a to be determined normalization factor between the number of points scored and the resulting percentage mark.

Copying results is not allowed. It's OK to ask for help from your friends. However, this help must not extend to copying code or written text that your friend has written, or that you and your friend have written together. I assess you on the basis of what you are able to do by yourself. It's OK to help a friend. However, this help must not extend to providing your friend with code or written text. If you are found to have done so, a penalty will be assessed against you as well.

Email me the Matlab script that you used for question 5, I will not assess the programming style, but I might check it if results are unexpected. I can also run plagiarism detectors on them. Email it to mvanross@inf.ed.ac.uk and the subject should contain 'nc2-2006' (all lowercase).

Deadline is Thursday 20 April at noon (standard late policies apply). Hard-copies preferred, but if you are out of town an email to me is ok (pdf or postscript format). Hand in to Pat Ferguson, Rm. D10 in Forrest Hill.

Model

We study learning rules that describe the weight change in terms of firing rates. The particular learning we explore here is that for every stimulus presentation $w_i \to w_i + \Delta w_i$,

$$\Delta w_i = \eta y (x_i - w_i)$$

where η is a small number (take $\eta = 10^{-3}$ throughout the simulations), x_i is the presynaptic input from neuron i, and w_i is the weight associated to that input. We write the weights as a vector $\mathbf{w}$. The postsynaptic activity y is initially simply modelled as $y = \mathbf{w}.\mathbf{x}$.

Question 1 (5 points) We are interested in the steady state weight vector. Assume that we present a sequence of inputs x_i^μ drawn from a certain distribution $P(x_1, x_2, \ldots)$. η is chosen so small that the weights change only little for a given input. Stimuli are presented until the weights no longer change on average. Give the expression that the steady state weight vector $\mathbf{w_{ss}}$ obeys. Show that $\mathbf{w}_{ss}$ is an eigenvector of the stimulus correlation matrix.

Question 2 (5 points) Simulate a single neuron receiving two inputs, drawn from a Gaussian distribution with

$$P(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^2 \det C}} \exp[-\frac{1}{2}(\mathbf{x}^T - \mathbf{x}_0^T) C^{-1} (\mathbf{x} - \mathbf{x}_0)]$$

where $C = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$ is the covariance matrix. The mean input is denoted $\mathbf{x_0}$ and is taken to be (0.1,0.1) throughout this assignment. What is the correlation matrix is this case? Start from different initial values for the weight and simulate for a sequence of about 10000 randomly drawn stimuli. What is/are the steady state weight vector(s)? Compare your results to question 1.

Note: to simulate correlated variables with a certain covariance matrix you can use the Cholesky decomposition of the covariance matrix.

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octave:> help chol
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Check your implementation by extracting the covariance matrix and means from your simulation.

Question 3 (5 points) Next consider the case that the inputs are anti-correlated with covariance matrix $C = \begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix}$. Simulate this case and explain the behaviour.

Question 4 (10 points) Consider the situation that the neuron, receives just one input, x, but also makes a synapse onto itself, so that $y = w_f x + w_r y$. Both the feedforward weight w_f and the recurrent weight w_r are subject to the plasticity rule. Plot the value of w_r against input strength x. Compare your simulation to theory.

From now on we restrict the incoming weights to be positive. We consider a network of 20 postsynaptic neurons. There are still only two inputs. The neurons have lateral interaction with local excitation and global inhibition. In order to update the weights we need to first calculate the postsynaptic activity given the input. The postsynaptic activity is modelled as $\tau \frac{d\mathbf{y}}{dt} = -\mathbf{y} + [W\mathbf{x} + L\mathbf{y}]_+$, where W is the weight matrix of the inputs. For every stimulus presentation let the activity $\mathbf{y}$ settle, before updating the weights. The value of τ is arbitrary.

Lateral interaction matrix $L_{ij} = l(i,j)(1 - \delta_{ij})$, $l(i,j) = 0.1$ if there are less or equal than 4 nodes between i and j, while $l(i,j) = -0.05$ otherwise. These lateral connections are kept fixed. Impose wrap-around boundary conditions, so that neuron 1 is a neighbour of neuron 20.

Question 5 (10 points) Implement the network. Consider the case that the inputs are anti-correlated with covariance matrix $C = \begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix}$. Again simulate until a steady state of the weights is reached. Plot the weights of the postsynaptic neurons versus their position in the network. Repeat for the case that the inputs are positively correlated with covariance matrix $C = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. Explain your observations.

Question 6 (10 points) Explore the behaviour of the resulting weights for various settings of the function $l(i,j)$ and both settings of the covariance matrix. Relate your finding to biology.