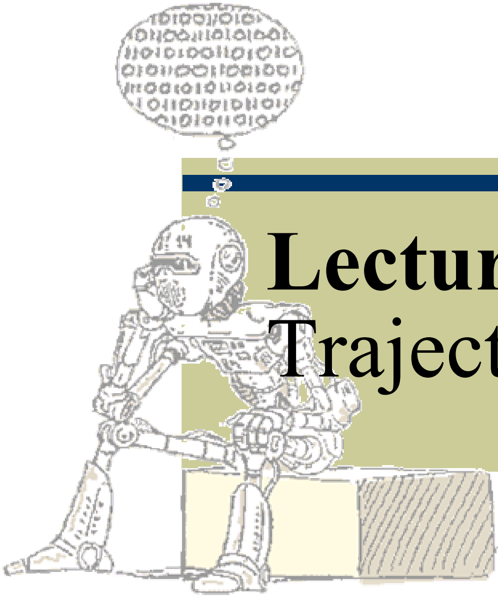


Lecture XIV
Trajectory Formation through Optimization

Contents:

- Optimization Criterion
  - Minimum Distance
  - Minimum Time
  - Minimum Acceleration Change
  - Minimum Torque Change
  - Minimum End Point Variance
- Using Motor Redundancies Efficiently

Trajectory Planning Phases

- **Trajectory generation**
  - Involves computation of the best trajectory for the object
- **Force Distribution**
  - Involves determining the force distribution between different actuators (a.k.a. resolving actuator redundancy)

Some of the approaches solve the trajectory generation and force distribution problems separately in two phases.

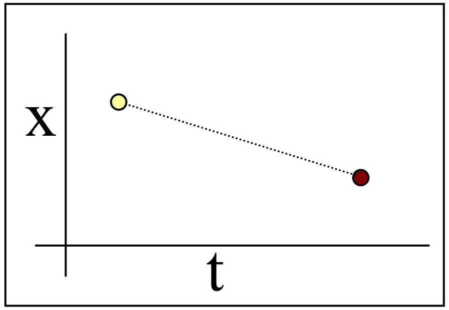
It has been argued that solving the two issues simultaneously (as a global optimization problem) is superior in many cases.

Kinematics: refers to geometrical and time-based properties of motion; the variables of interest are positions (e.g. joint angles or hand Cartesian coordinates) and their corresponding velocities, accelerations and higher derivatives.

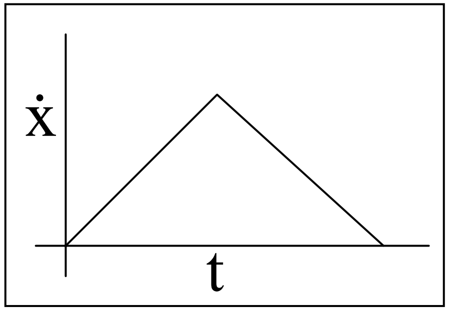
Dynamics: refers to the forces required to produce motion and is therefore intimately linked to the properties of the object such as it's mass, inertia and stiffness.

Some Simple Cost Functions

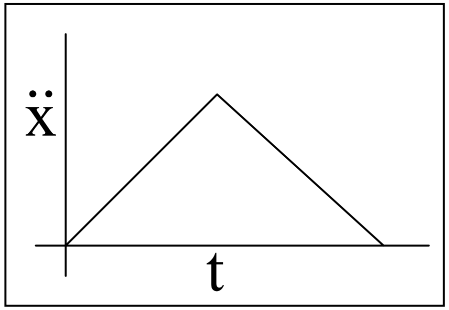
- **Shortest Distance**
  - Refer: *F.C.Park and R.W.Brockett, Kinematic Dexterity of robot mechanisms, Int. Journal of Robotic Research, 1391), pp. 1-15, 1994*



- **Minimum Acceleration**
  - Refer: *L. Noakes, G. Heinzinger, and B. paden, Cubic splines on curved surfaces, IMA Journal of Mathematical Control & Information, 6, pp. 465-473, 1989.*



- **Minimum Time (Bang-Bang Control)**
  - Refer: *Z. Shiller, Time Energy optimal control of articulated systems with geometric path constraints, In. Proc. of 1994 Intl. Conference on Robotics and Automation, pp. 2680-2685, San Diego, CA, 1994.*



Minimum Jerk Trajectory Planning

- **Proposed by Flash & Hogan (1985)**
- **Optimization Criterion minimizes the jerk in the trajectory**

$$C_J = \frac{1}{2}\int_0^T \left(\left(\frac{d^3x}{dt^3} \right)^2 + \left(\frac{d^3y}{dt^3} \right)^2 \right) dt$$

For movement in x-y plane

- **The *minimum-jerk* solution can be written as:**

$$x(t) = x_0 + (x_0 - x_f)(15\hat{t}^4 - 6\hat{t}^5 - 10\hat{t}^3)$$

$$y(t) = y_0 + (y_0 - y_f)(15\hat{t}^4 - 6\hat{t}^5 - 10\hat{t}^3)$$

$$where \quad \hat{t} = t / t_f \;\; and \;\; (x_0, y_0) \; are \; initial \; coordinates \; at \; t = 0.$$

- Depends only on the kinematics of the task and is independent of the physical structure or dynamics of the plant
- Predicts bell shaped velocity profiles

Minimum Torque Change Planning

- **Proposed by Uno, Kawato & Suzuki (1989)**
- **Optimization Criterion minimizes the change of torque**

$$C_T = \frac{1}{2}\int_0^T \left(\left(\frac{d\tau_1}{dt} \right)^2 + \left(\frac{d\tau_2}{dt} \right)^2 \right) dt$$

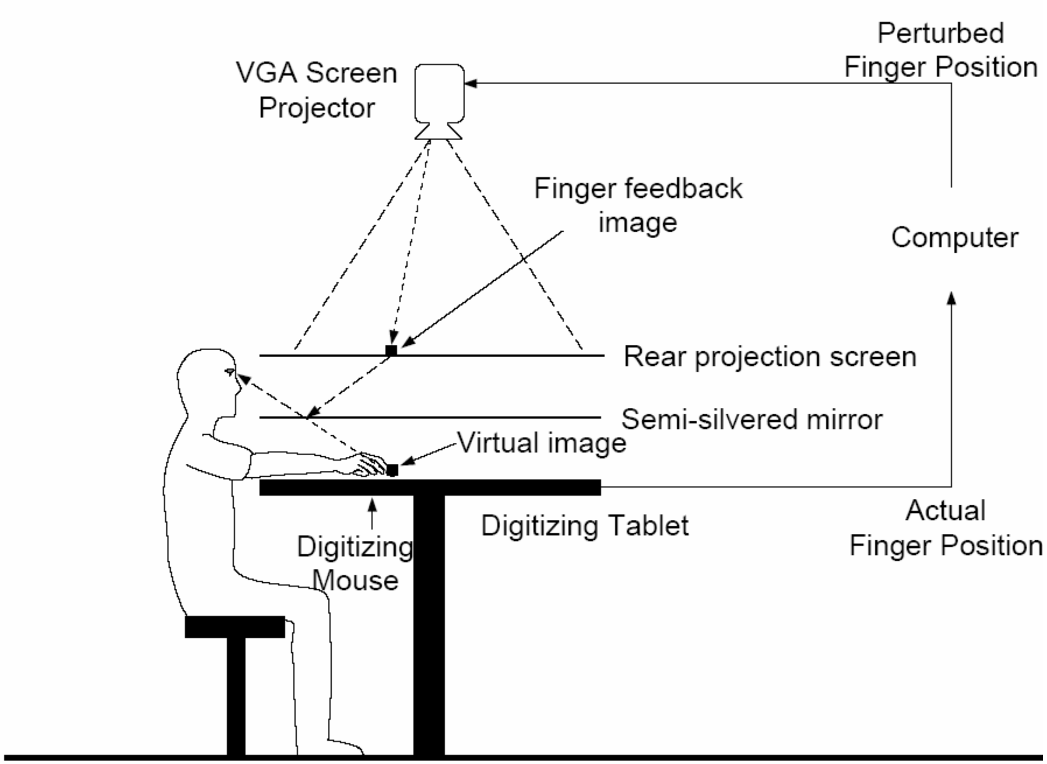
For two joint arm or robot

- **The Min. Jerk and Min. Torque change cost functions are closely related since acceleration is proportional to torque at zero speed.**
- No Analytical solution possible for Min. Torque change criterion but iterative solution is possible.

- Like Min. Jerk, predicts bell shaped velocity profiles.
- But also predicts that the form of the trajectory should vary across the arm's workspace.

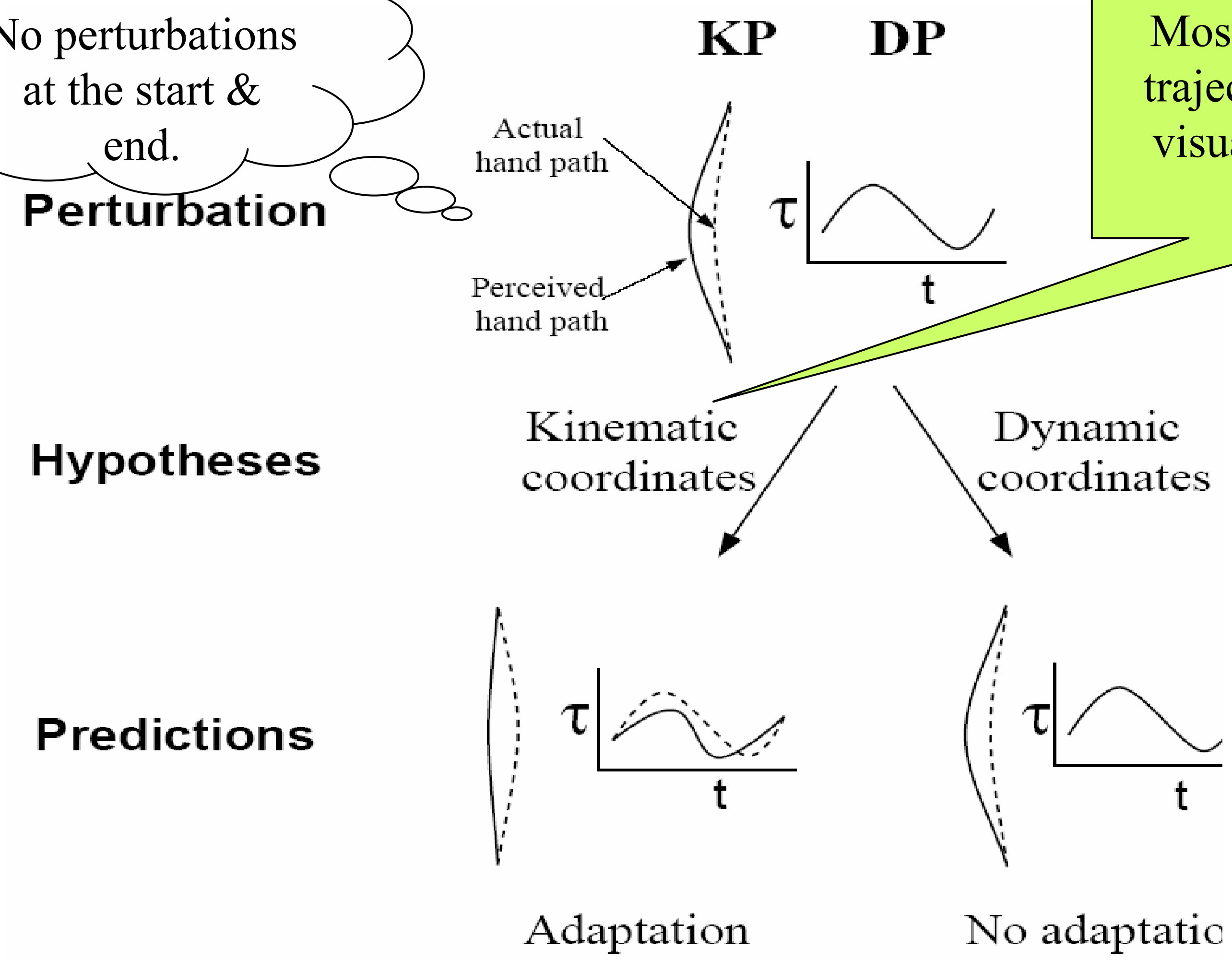
Min. Jerk vs Min. Torque Change

One way of resolving how humans plan their movement is by setting up a experiment which can distinguish between the kinematic vs dynamic plans



Min. Jerk vs Min. Torque Change (II)

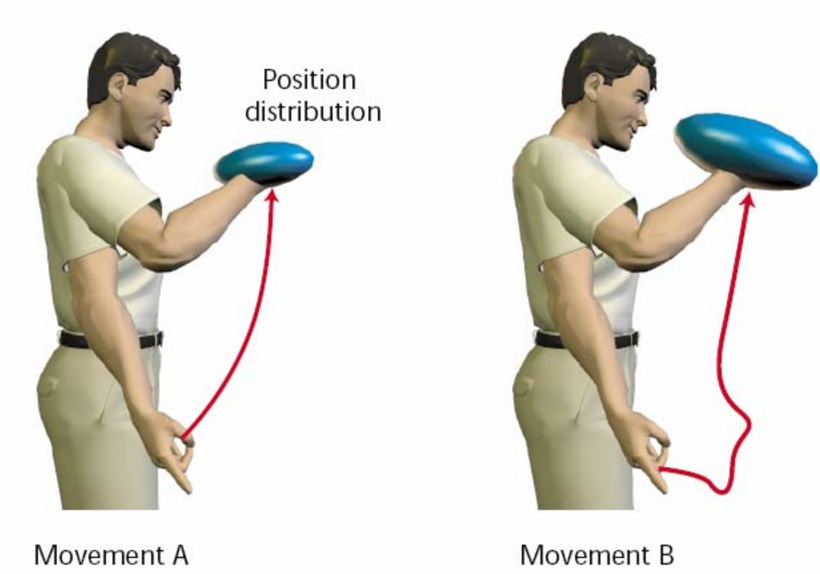
No perturbations at the start & end.



Most studies suggest that trajectories are planned in visually-based kinematic coordinates

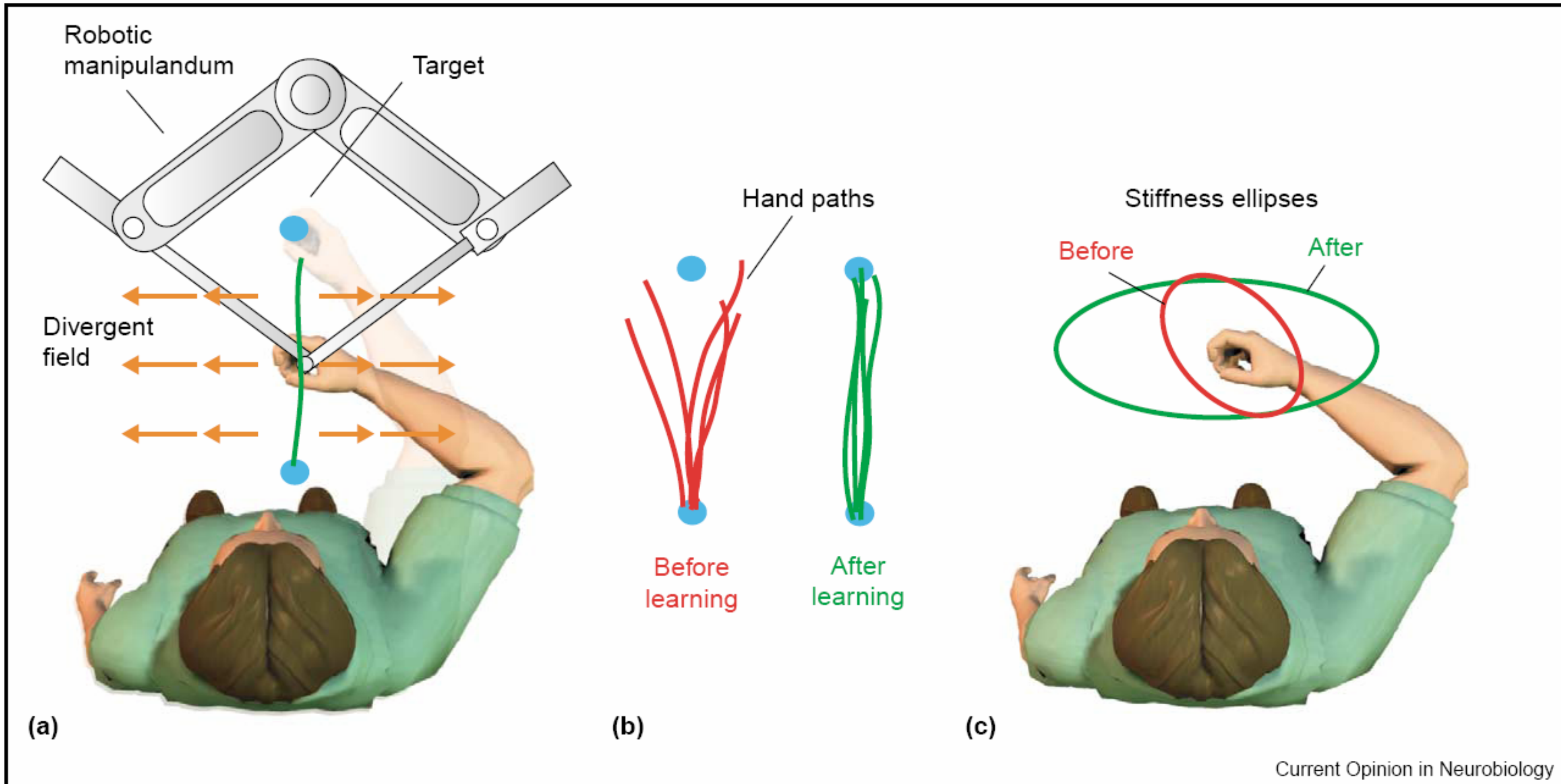
Minimum Endpoint Variance Planning

- **Proposed by Harris & Wolpert (1998)**
- **Also called TOPS (Trajectory Optimization in the Presence of Signal dependent noise)**
- **Basic Theory:**
  - Single physiological assumption that neural signals are corrupted by noise whose variance increases with the size of the control signal.
  - In the presence of such noise, the shape of the trajectory is selected to minimize the variance of the final end-point position.
  - Biologically more plausible since we do have access to end point errors as opposed to complex optimization processes (min. jerk and min. torque change integrated over the entire movement) that other optimization criterion suggest the brain has to solve.



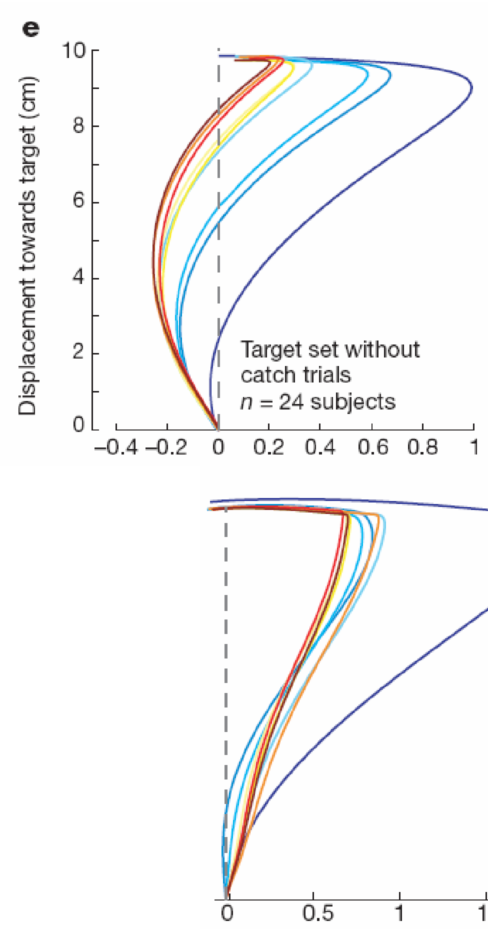
Refer: *Harris & Wolpert, Signal-dependent noise determines motor planning, Nature, vol. 394, 780-784*

Testing the Internal Model Learning Hypothesis

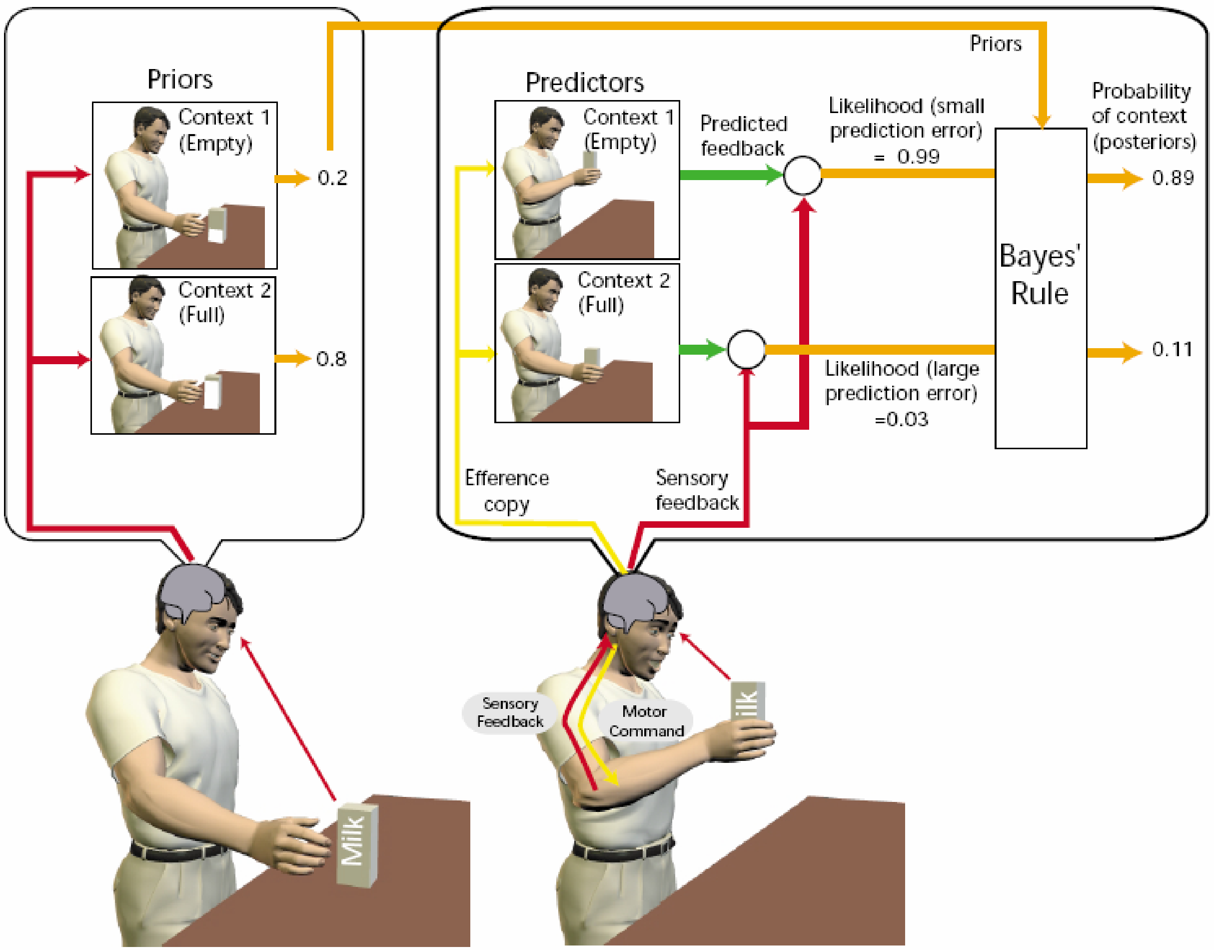


Learning of Internal Models

- Using the force manipulandum, one can create an interesting experiment in which you modify the dynamics of your arm movement in only the x-y plane.
- Humans are very adept at learning these changed dynamic fields and adapt at a relatively short time scale.
- When we remove these effects, after effects of learning can be felt for some time before re-adaptation, providing evidence that we learn internal models and use them in a predictive feedforward fashion

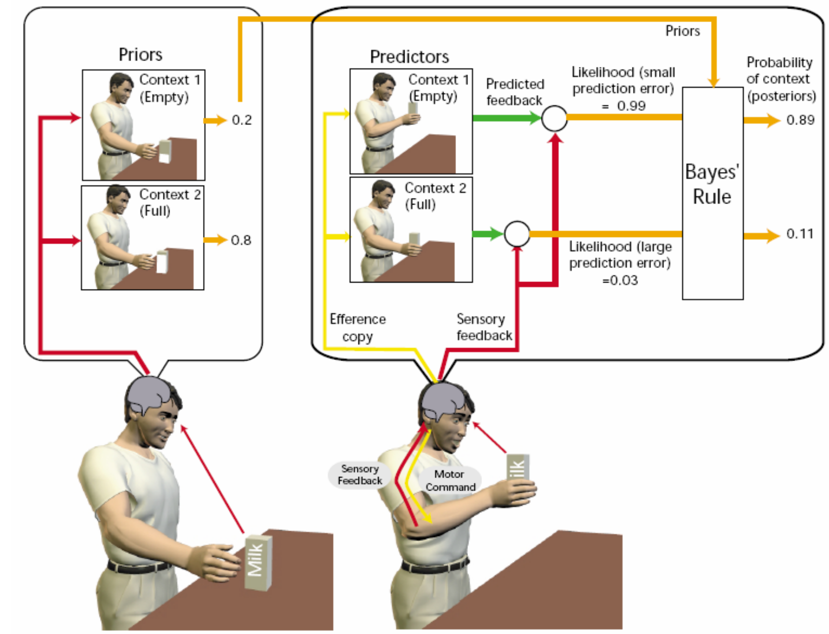


Multiple Model Hypothesis



Recursive Bayes Estimation

$$P(y \mid x) = \frac{P(x \mid y)P(y)}{P(x)}$$



If we explicitly index the individual experiences by n, i.e., $Đ^n = \{x_1, \ldots, x_n\}$,

Using …

$$p(D \mid \theta) = \prod_{k=1}^{n} p(\mathbf{x}_k \mid \theta)$$

$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\int p(D \mid \theta)\, p(\theta) d\theta}$$

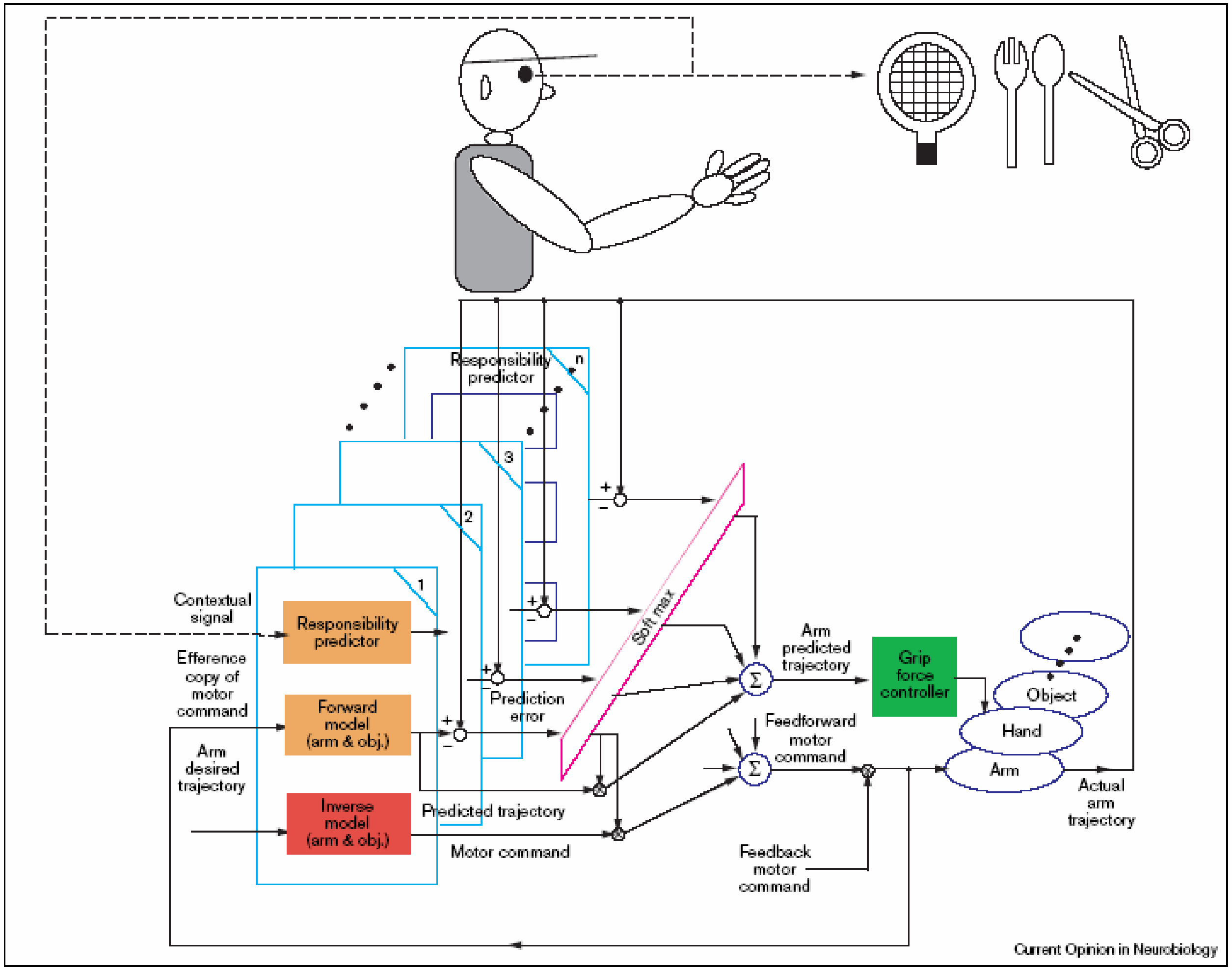
We get …

$$p(D^n \mid \theta) = p(\mathbf{x}_n \mid \theta)\, p(D^{n-1} \mid \theta)$$

$$p(\theta \mid D^n) = \frac{p(\mathbf{x}_n \mid \theta)\, p(\theta \mid D^{n-1})}{\int p(\mathbf{x}_n \mid \theta)\, p(\theta \mid D^{n-1}) d\theta}$$

$$where \quad p(\theta \mid D^0) = p(\theta)$$

Multiple model hypothesis (II)



Multiple Paired Forward-Inverse Models (MPFIM)

Resolving Motor Redundancies

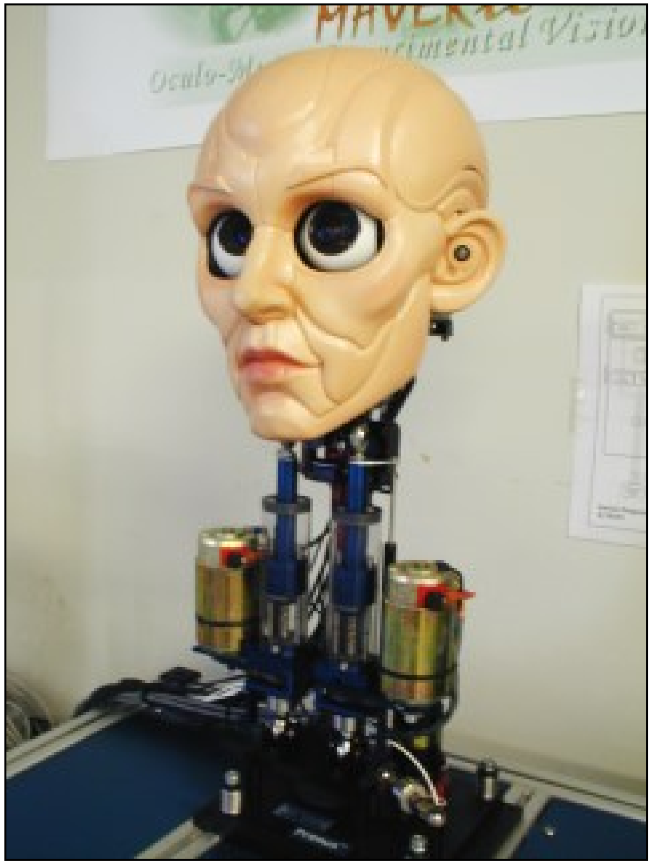
Inverse Kinematics

$$(\Delta \mathrm{x}) \to (\Delta \theta) \quad f_{eye+head} : \Re^4 \to \Re^7$$

Retinal Displacement

$$\mathbf{x} = \left(x_R \; x_L \; y_R \; y_L\right)^T$$
$$\boldsymbol{\theta} = \left(\theta_{Head\,1} \theta_{Head\,2} \theta_{Head\,3} \; \theta_{EyeLP} \; \theta_{EyeLT} \; \theta_{EyeRP} \; \theta_{EyeRT}\right)^T$$

Eye $ Head displacements



Resolving kinematics with RMRC

Resolved Motion Rate Control with locality in joint positions

$$\Delta x = \mathbf{J}(\theta)\, \Delta\theta$$

Integrate over small increments (*where linearity holds*) to get complete trajectory

Based on collected training data forward kinematics in velocity space was almost completely linear, irrespective of joint position

$$\dot{x} = \mathbf{J}\, \dot{\theta}$$

Hence, in this case, we can use pseudo-inversion with the *constant* Jacobian !!

Pseudoinverse & Null Space manipulation

$$Given\ \ \dot{\mathrm{x}} = \mathbf{J}\,\dot{\theta}\,,\ what\ is\ \ \dot{\theta}\,?$$

$$\dot{\theta} = \mathbf{J}^{\#}\dot{\mathrm{x}} + (\mathrm{I} - \mathbf{J}^{\#}\mathbf{J})\,\mathbf{k}_{null}$$

Optimization based on gradient descent in Null space

$$\mathbf{J}^{\#} = \mathbf{J}^{T}(\mathbf{J}\mathbf{J}^{T})^{-1}$$ **: Pseudoinverse**

$$k_{null,i} = -\frac{\partial L_{opt}}{\partial \theta_{current,i}} = w_i(\theta_{current,i} - \theta_{default,i})$$

Inertial Weighting for each joint

$$L_{opt} = \min \frac{1}{2}\sum w_i(\theta_{current,i} - \theta_{default,i})^2$$ *Optimization criterion*