Bayesian update with basis functions

Prior: \[ p(w) = \mathcal{N}(w; w_0, V_0) \]

Prior also includes model choices \( \phi \)

\[ V_N = \sigma_y^2 (\sigma_y^2 V_0^{-1} + \Phi^T \Phi)^{-1} \]

\[ w_N = V_N V_0^{-1} w_0 + \frac{1}{\sigma_y^2} V_N \Phi^T y \]

Bayes’ Rule + data

Posterior: \[ p(w|D) = \mathcal{N}(w; w_N, V_N) \]

Likelihood:

\[ p(y|x, w) = \mathcal{N}(y; \Phi w, \sigma_y^2) \]

2019 L12 (1)
“Underfitting”

No basis functions
Simple model
\Rightarrow over-confident
Residuals
Correlations of Residuals
\Rightarrow Model checking

Overfitting

The Bayesian Method does not involve fitting.
\Rightarrow Don't overfit

But...
Extremely flexible model

\[ 10^6 \text{ basis functions (RBFs)} \]

Can represent the function

\[ f(x) \]

prior: \( p(w_k) = \mathcal{N}(w_k; 0, \sigma_k^2) \)

for each function \( k \)

\( k = 1, \ldots, 10^6 \)

\[ p(w) = \mathcal{N}(w; 0, \sigma^2 \mathbf{I}) \]

\( \sigma^2 \) big matrix

2019 L12 ③
Posterior for the flexible model
Probabilistic model choice

E.g. Bayes classifiers

\[ p(x | M_1) \]

\[ p(x | M_2) \]

Test point \( x \)

Model for class 1

Model for class 2

Used likelihood of model fitted

Model choice for regression

\[ p(y | X, M) \]

\[ \sum \text{All training inputs} \]

\[ \text{All training labels} \]

"Marginal likelihood"
or just "likelihood" of the model

Could apply:

1. Bayes' rule \[ \Rightarrow \quad p(M | y, X) \]

2. Maximise likelihood

2019 L12 ⑤
\[ p(y | X, M) = \int p(y, w | X, M) \, dw \]

\[ = \int p(y | X, w, M) \, p(w | X, M) \, dw \]

"Standard" likelihood

Prior

Trick to solve integral:

Posterior weights:

\[ p(w | y, X, M) = \frac{p(y | w, X, M) \cdot p(w | X, M)}{p(y | X, M)} \]

Rearrange and evaluate
Example model choice

100 basis functions, spaced between 0, 1

Model 1: Bandwidth $h = 1$

Model 2: $h = 0.01$

\[ p(y | x, h=1) \gg p(y | x, h=0.01) \]

\[ p(y | x, h=0.01, \hat{w}) \text{ big} \]