# Recurrent Neural Networks 1: Modelling sequential data

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Machine Learning Practical — MLP Lecture 9
12 November 2019

# Sequential Data






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- We often wish to model data that is a sequence or trajectory through time, for instance, text (sequences of characters/words), audio signals, currency exchange rates, motion of a vehicle
- What should a good model of sequential data include?

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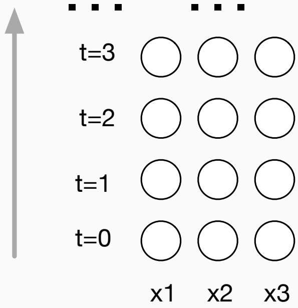
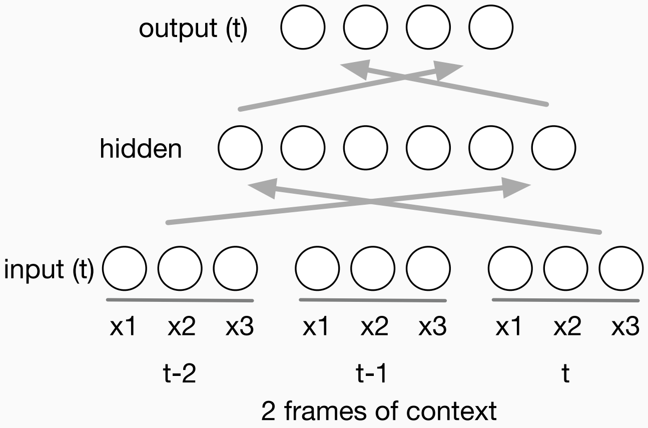
- We often wish to model data that is a sequence or trajectory through time, for instance, text (sequences of characters/words), audio signals, currency exchange rates, motion of a vehicle
- What should a good model of sequential data include?
  - Track long-term dependencies
    Eg "Hinton is one of the fathers of deep learning. He was co-author of a highly cited paper ..."
  - Learn invariances across time
    Eg "I went to Nepal in 2009" and "In 2009, I went to Nepal"
  - Handle variable-length sequences
- Convolutional networks model invariances across space – can we do something similar to model invariances across time?
  - Yes - time-delay neural networks

# Modelling sequences




- Imagine modelling a time sequence of 3D vectors

# Modelling sequences




- Imagine modelling a time sequence of 3D vectors
- Can model fixed context with a feed-forward network with previous time input vectors added to the network input

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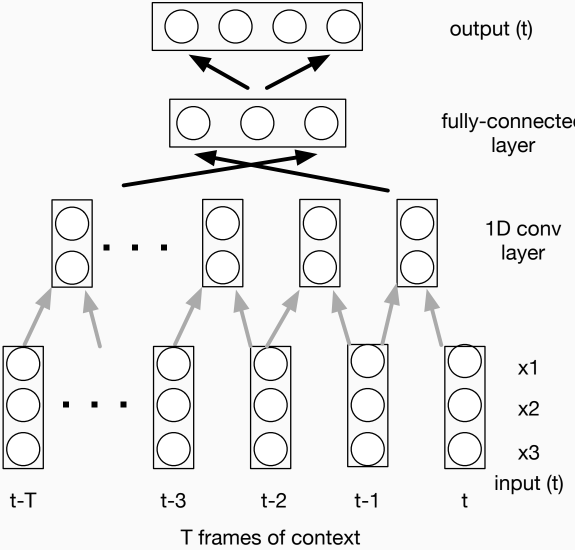


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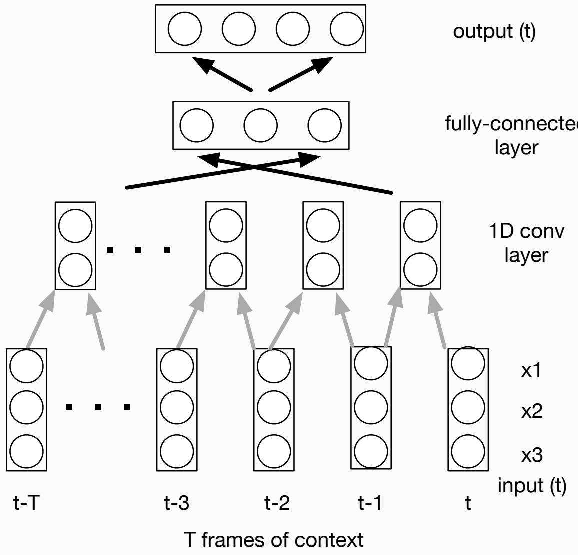
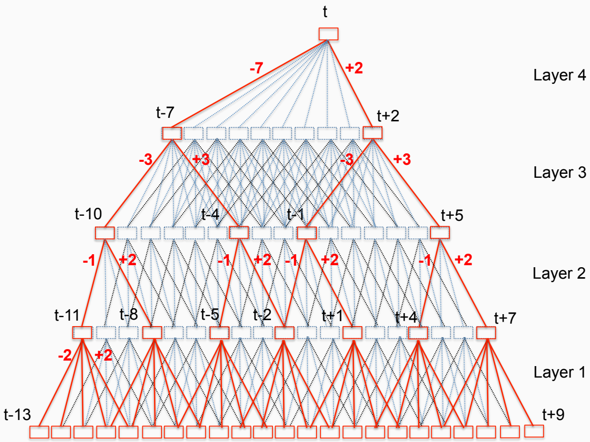
- Model using 1-dimension convolutions in time - **time-delay neural network** (TDNN)

# Modelling sequences




- Imagine modelling a time sequence of 3D vectors
- Can model fixed context with a feed-forward network with previous time input vectors added to the network input
- Model using 1-dimension convolutions in time - **time-delay neural network** (TDNN)
- Network takes into account a *finite context*

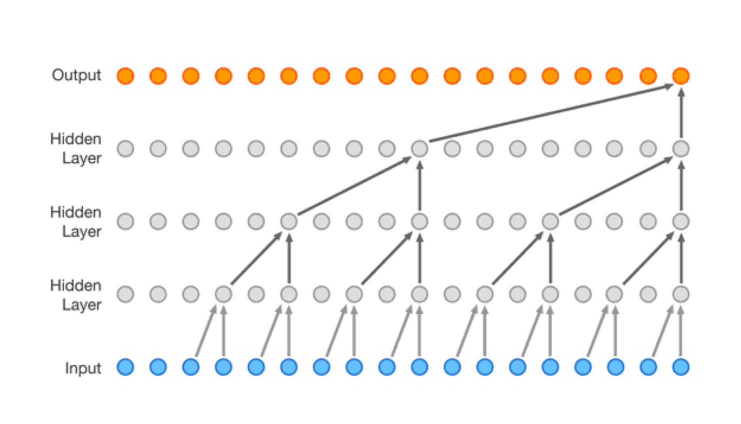
# TDNNs in action – Speech-to-text




- TDNN operating on 23 frames of context
- Without sub-sampling (blue+red)
- With sub-sampling (red)

Peddinti et al, " Reverberation robust acoustic modeling using i-vectors with time delay neural networks", Interspeech-2015, http://www.danielpovey.com/files/2015_interspeech_aspire.pdf

# Wavenet




van den Oord et al (2016), “WaveNet: A Generative Model for Raw Audio”, https://arxiv.org/abs/1609.03499

# RNNs vs CNNs

- CNNs can scale to images with large width and height
- Some CNNs can process images of variable size

# RNNs vs CNNs

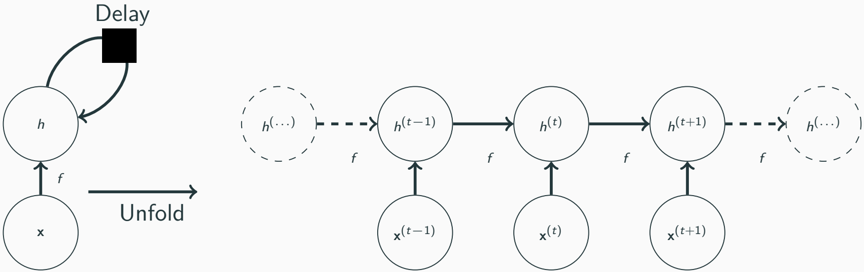
- CNNs can scale to images with large width and height
- Some CNNs can process images of variable size
- RNNs can scale to much longer sequences than would be practical for networks without sequence-based specialization
- RNNs can also process sequences of variable length

# Dynamical systems

$$s^{(\dots)} \xrightarrow{f} s^{(t-1)} \xrightarrow{f} s^{(t)} \xrightarrow{f} s^{(t+1)} \xrightarrow{f} s^{(\dots)}$$

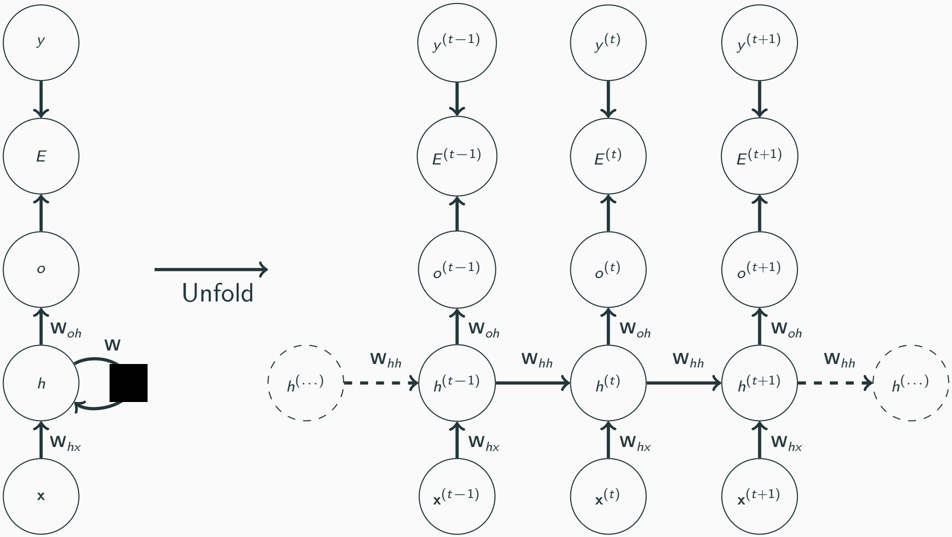
- State of the system $s^{(t)} = f(s^{(t-1)}; \mathbf{W})$
- Unfolding $s^{(3)} = f(s^{(2)}; \mathbf{W}) = f(f(s^{(1)}; \mathbf{W}); \mathbf{W})$
- Assumption: The rules of the universe that govern the dynamic system does not change over time
- Property 1: Regardless of the sequence length, the learned model always has the same input size
- Property 2: It uses the same transition function $f$ with the same weights $\mathbf{W}$ at every time step

# Unfolding computational graphs

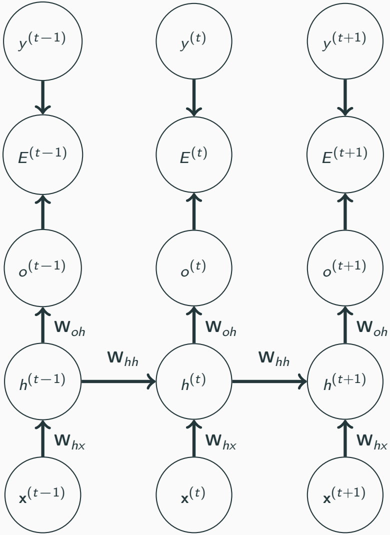



- Observation as input $\mathbf{x}$
- State units are not visible, now hidden $h^{(t)} = f(h^{(t-1)}, x^{(t)}; \mathbf{W})$
- State units as memory – remember things for (potentially) an infinite time
- State units as information compression – compress the history (sequence observed up until now) into a state representation

# Simple RNN: Recurrent hidden units



# Simple RNN with recurrent hidden unit




- Hidden state $h^{(t)}$

$$h^{(t)} = \tanh\left(\mathbf{W}_{hx}\mathbf{x}^{(t)} + \mathbf{W}_{hh}\mathbf{h}^{(t-1)}\right)$$

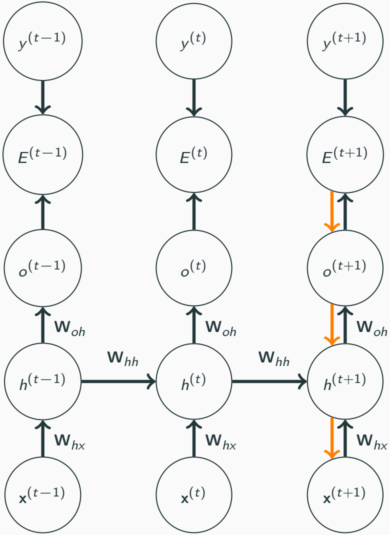
- Output $o^{(t)}$

$$o^{(t)} = \mathrm{softmax}\left(\mathbf{W}_{oh}\mathbf{h}^{(t)}\right)$$

- Error $E^{(t)}$ for ground-truth $y^{(t)}$

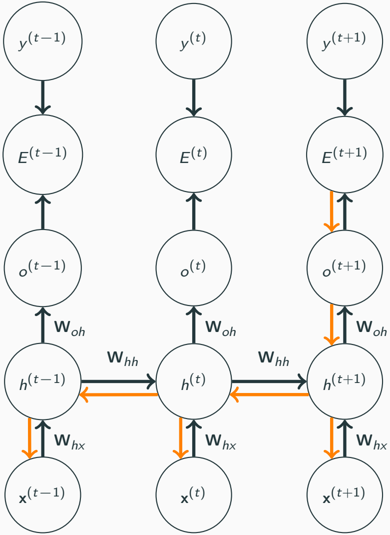
$$E^{(t)} = CE\left(o^{(t)}, y^{(t)}\right)$$

# Training simple RNN with recurrent hidden unit




- View an RNN for a sequence of $\tau$ inputs as a $\tau$-layer network with shared weights
- Train an RNN by doing backprop through this unfolded network

# Training simple RNN with recurrent hidden unit




$$\frac{\partial E^{(t+1)}}{\partial \mathbf{W}_{oh}} = \frac{\partial E^{(t+1)}}{\partial o^{(t+1)}} \frac{\partial o^{(t+1)}}{\partial \mathbf{W}_{oh}}$$

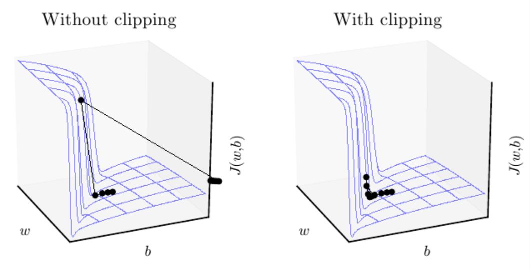
$$\frac{\partial E^{(t+1)}}{\partial \mathbf{W}_{hx}} = \frac{\partial E^{(t+1)}}{\partial o_{(t+1)}} \frac{\partial o^{(t+1)}}{\partial h^{(t+1)}} \Big( \frac{\partial h^{(t+1)}}{\partial \mathbf{W}_{hx}} + \frac{\partial h^{(t+1)}}{\partial h^{(t)}} \frac{\partial h^{(t)}}{\partial \mathbf{W}_{hx}} + \frac{\partial h^{(t+1)}}{\partial h^{(t)}} \frac{\partial h^{(t)}}{\partial h^{(t-1)}} \frac{\partial h^{(t-1)}}{\partial \mathbf{W}_{hx}} + \ldots \Big)$$

$$\frac{\partial E^{(t+1)}}{\partial \mathbf{W}_{hh}} = \frac{\partial E^{(t+1)}}{\partial o^{(t+1)}} \frac{\partial o^{(t+1)}}{\partial h^{(t+1)}} \Big( \frac{\partial h^{(t+1)}}{\partial \mathbf{W}_{hh}} + \frac{\partial h^{(t+1)}}{\partial h^{(t)}} \frac{\partial h^{(t)}}{\partial \mathbf{W}_{hh}} + \frac{\partial h^{(t+1)}}{\partial h^{(t)}} \frac{\partial h^{(t)}}{\partial h^{(t-1)}} \frac{\partial h^{(t-1)}}{\partial \mathbf{W}_{hh}} + \ldots \Big)$$

# Back-propagation through time (BPTT)

- We can train a network by unfolding and *back-propagating through time*, summing the derivatives for each weight as we go through the sequence
- More efficiently, run as a recurrent network
  - cache the unit outputs at each timestep
  - cache the output errors at each timestep
  - then backprop from the final timestep to zero, computing the derivatives at each step
  - compute the weight updates by summing the derivatives across time
- Expensive – backprop for a 1,000 item sequence equivalent to a 1,000-layer feed-forward network
- Truncated BPTT – backprop through just a few time steps (e.g. 20)

# Gradient clipping



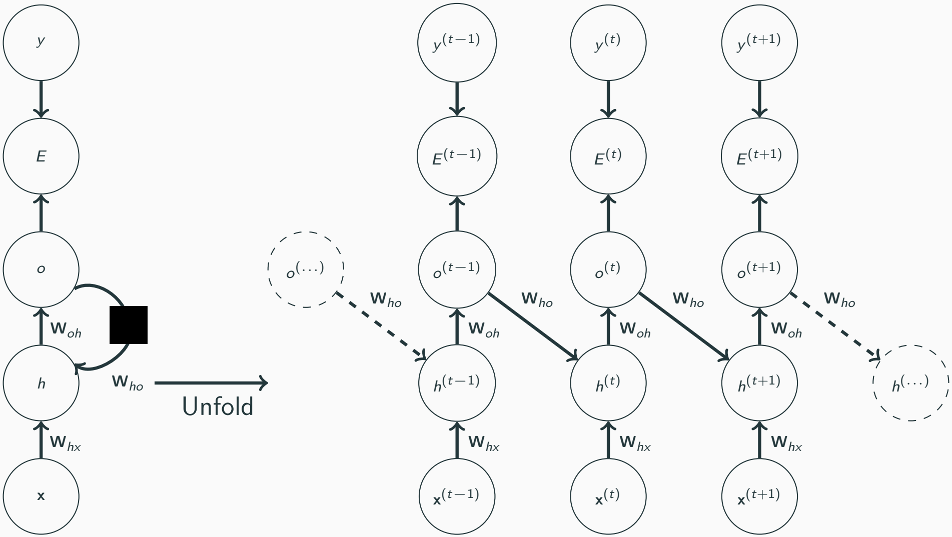

- Gradient clipping can make gradient descent perform more reasonably in the vicinity of extremely steep cliffs

$$\text{if } \|g\| > v, \text{ then } g \leftarrow \frac{vg}{\|g\|}$$

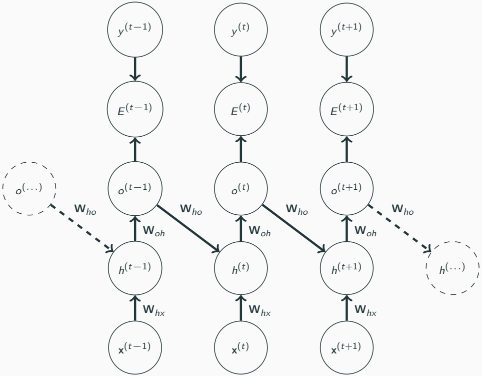
- Can be used with RMS-Prop and Adam

Image credit: Goodfellow et al. 2016

# Recurrence only through the output



# Recurrence only through the output

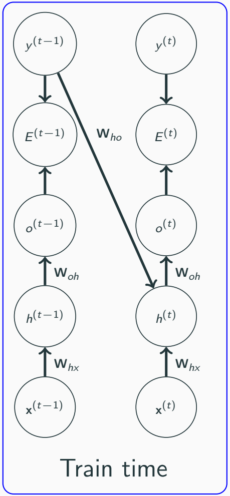


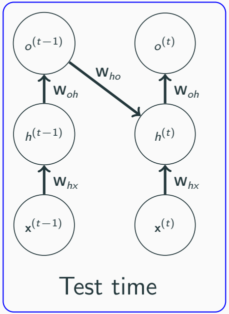

- The feedback connection is from the output to the hidden layer

$$\mathbf{h}^{(t)} = \tanh\left(\mathbf{W}_{hx}\mathbf{x}^{(t)} + \mathbf{W}_{ho}\mathbf{h}^{(t-1)}\right)$$

- More restricted hidden recurrent network, $o$ is the only information it is allowed to send to the future

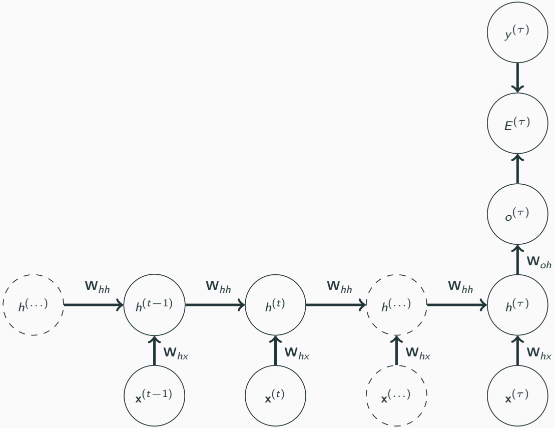
# Teacher forcing in output recurrent network



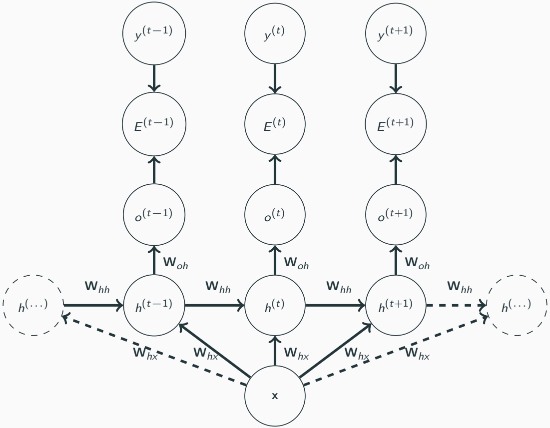

- Teacher forcing: the model receives the ground truth output $y^{(t)}$ during training
- Each loss function at time $t$ is decoupled
- Training can thus be parallelized: the gradient for each step $t$ computed in isolation
- Disadvantage: Train and test time difference
- One solution is to train with both teacher-forced inputs ($y$) and free-running inputs ($o$)

# Sequence input, single output ("seq2vec")




- RNN with a single output at the end of the sequence
- Summarize a sequence and produce a xed-size representation used as input for further processing
- Eg sentiment analysis

# Single input, sequence output ("vec2seq")

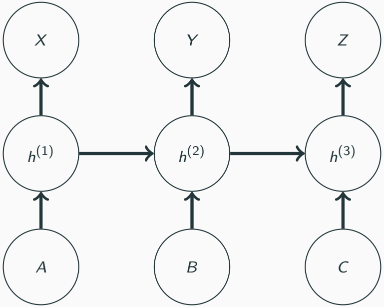



- RNN with single input and sequence output
- Each element $y^{(t)}$ of the observed output sequence serves both as input (for the current time step) and, during training, as target (for the previous time step)
- Eg image captioning

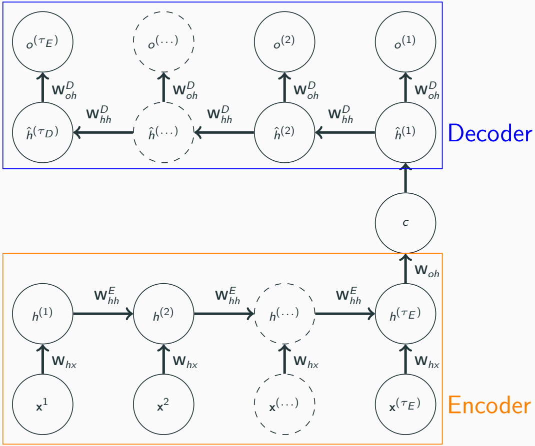
# Question

We want to translate a sentence from one language to another one.

Is the architecture below suitable for this problem?



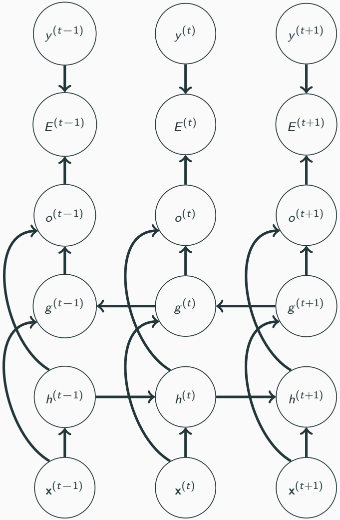
# (Vanilla) sequence to sequence model ("seq2seq")




- seq2seq = seq2vec + vec2seq
- Encoder takes in a sequence of inputs and compresses it into a context vector $c$
- Decoder takes in a context vector and outputs a sequence
- Eg machine translation

Sutskever, et al, "Sequence to sequence learning with neural networks." NeurIPS2014.

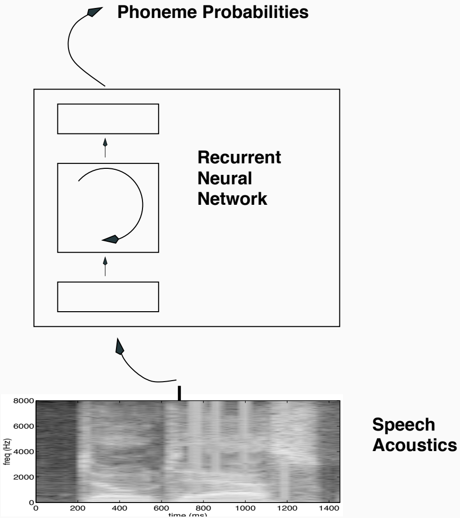
# Bidirectional RNN




- So far, "causal" relation, the state at time $t$ captures only information from the past, $x^{(1)}, \ldots, x^{(t-1)}$ and the present input $x^{(t)}$
- In some applications, we want the output $o^{(t)}$ to depend on the whole sequence
- Bidirectional RNN – combine an RNN moving forward in time, with one moving backwards in time
- State units provide a combined representation that depends on both the past and the future

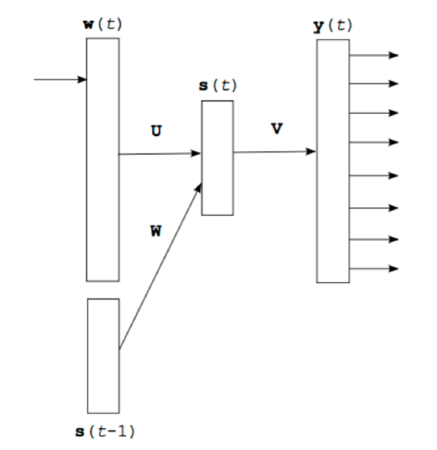
# Recurrent neural networks examples from ancient history (using "vanilla" RNNs and BPTT)

# Example 1: speech recognition with recurrent networks




T Robinson et al (1996).
"The use of recurrent networks in continuous speech recognition",
in *Automatic Speech and Speaker Recognition Advanced Topics*
(Lee et al (eds)), Kluwer, 233–258.
http://www.cstr.ed.ac.uk/downloads/publications/1996/rnn4csr96.pdf

# Example 2: recurrent network language models




T Mikolov et al (2010).
"Recurrent Neural Network Based Language Model",
*Interspeech*
http://www.fit.vutbr.cz/research/groups/speech/publi/2010/mikolov_interspeech2010_IS100722.pdf

# Summary

- Model sequences using finite context using feed-forward networks with convolutions in time (TDNNs, Wavenet)
- Model sequences using infinite context using recurrent neural networks (RNNs)
- Unfolding an RNN gives a deep feed-forward network with shared weights
- Train using back-propagation through time
- (Historical) examples on speech recognition and language modelling
- Reading: Goodfellow et al, chapter 10 (sections 10.1, 10.2, 10.3) http://www.deeplearningbook.org/contents/rnn.html
- Next lecture: LSTM, sequence-to-sequence models, attention