Music Informatics

Alan Smaill

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Today

- WTM pitch and key
- Tuning systems
- A basic key analysis algorithm
Just as most WTM is organised around a regular metrical hierarchy, the pitches are typically organised around a standard set of (relative) pitches, where:

- the interval of the octave plays a key role: notes are named (like g#) independently of the octave, voices singing an octave apart are singing “in unison”.
- the intermediate notes are spaced roughly equally between the octave, as on piano keyboard or guitar frets.

This doesn’t stop other pitches appearing, for expressive or other purposes. But the claim is that pitches are related to such an underlying framework.
WTM is organised around a system of keys (like A major, B♭ minor).

A key is associated with a tonal centre (A) and a scale selected from the 12 semi-tones, in the major or minor pattern.

Notice that the names chosen for the notes on a keyboard are different depending on the key (g♯, a♭), and a system to turn a midi file into a conventional score has to work out the right spelling for the note. This depends on determining the key from a set of notes.
There are different ways in which the semitone pitches have been related to each other historically, going back to Pythagoras. There was a big change at the time of Bach which allowed keyboard instruments to play in “remote” keys (like F♯ major) and still sound “in tune”. For an overview of these different tunings, see eg

http://www.earmaster.com/fr/
music-theory-online/ch06/chapter-6-2

The perfect fifth (eg C – G) is realised as the interval between the second and third harmonics (of the note an octave below the C here); physically these notes have frequencies in the ratio 2:3. Singers will aim for this tuning.
Pythagorean vs mean vs equal tuning

In the Pythagorean tuning, major fifths are tuned exactly up and down from the tonic.

Pythagorean
Total Intervals:1 9:8 81:64 4:3 3:2 27:16 243:128 2
Note: C D E F G A B C
Cents: 204 204 90 204 204 204 90

The usual tuning before 1700 was just temperament (or mean tone), where major thirds are tuned exactly:

Mean/just
Total Intervals:1 5:4 2
Note: C D E F G A B C
Cents: 193 193 117 193 193 193 117
The equal temperament scale makes every semitone the same size. So tone/semitone have size 200/100 cents (by definition of cent).

<table>
<thead>
<tr>
<th>Note</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cents</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Comparison

It is not always easy to hear the difference between tunings; it is worth listening several times here. Some examples of just tuning with equal temperament are at

http://www.wmich.edu/mus-theo/groven/compare.html

Note that this has been set up so that just temperament is used even for a piece in C♯ major, by basing the tuning on that key (unlike in historic keyboard instruments).
Local key-finding in equal temperament

Problem: given a sequence of pitches in equal temperament (just as a semitone position on a semitone scale), estimate
- whether the melody is major or minor
- what the tonal centre is (keynote).

Longuet-Higgins developed a geometrical representation of semitone pitches in a 2-dimensional array that allows for multiple occurrences of the same (in equal temperament) pitches, depending on their role in different keys. The algorithm tries to match a given set of pitches to possible occurrences on the array, so as to have the occurrences as close together as possible. The shape of the occurrences then suggests major or minor, and the keynote.
major key by 3rds and fifths

The basis for the two dimensional array is
- intervals horizontally are perfect fifths (7 semitones)
- intervals vertically are major thirds (4 semitones)

For the C major scale, this gives

```
<table>
<thead>
<tr>
<th>A</th>
<th>E</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>C</td>
<td>G</td>
</tr>
</tbody>
</table>
```

and for (harmonic version of) C minor, get

```
<table>
<thead>
<tr>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
</tr>
<tr>
<td>A♭</td>
</tr>
</tbody>
</table>
```
The array

This pattern in the general array indicates major and minor scales, with the keynote at a distinguished position. Recall that the spelling of the note name depends on position in the array.

\[
\begin{array}{cccccccc}
E & B & F\# & C\# & G\# & D\# & A\# & E\# & B\#
\end{array}
\]

\[
\begin{array}{cccccccc}
C & G & D & A & E & B & F\# & C\# & G\#&
\end{array}
\]

\[
\begin{array}{cccccccc}
A_b & E_b & B_b & F & C & G & D & A & E
\end{array}
\]

\[
\begin{array}{cccccccc}
F_b & C_b & G_b & D_b & A_b & E_b & B_b & F & C
\end{array}
\]

This array can be extended, eg above and below where double sharp/double flat spellings are found.
The aim of the algorithm, given a sequence of pitches, is to look for a mapping of pitches into the array that corresponds to the major or minor key shape. This can be done computationally in different ways, eg. by positioning the key pattern at different places over the array, and seeing how many of the given notes can be mapped inside this pattern.
Example

Look at the subject of fugue 5, book 2 in D major. Given a midi-like representation of the notes, how do we work out whether the last note is F♯ or G♭?

There are 6 distinct pitches; the key is ambiguous between D major and G major, but in either case the last note is F♯.
Example ctd

For G major:

\[
\begin{align*}
F^\# & \quad C^\# & \quad G^\# & \quad D^\# & \quad A^\# & \quad E^\# & \quad B^\# \\
D & \quad A & \quad E & \quad B & \quad F^\# & \quad C^\# & \quad G^\# \\
B_b & \quad F & \quad C & \quad G & \quad D & \quad A & \quad E \\
G_b & \quad D_b & \quad A_b & \quad E_b & \quad B_b & \quad F & \quad C
\end{align*}
\]

This shape suggests a missing but implied C natural; G\textsubscript{b} is far away from the active pitches.
Example ctd

There is a similar story for D major:

\[
\begin{array}{cccccccc}
F\# & C\# & G\# & D\# & A\# & E\# & B\# \\
D & A & E & B & F\# & C\# & G\# \\
Bb & F & C & G & D & A & E \\
Gb & Db & Ab & Eb & Bb & F & C \\
\end{array}
\]

Now it is the C\# that is suggested (and not D\#); it is a different occurrence of E also.
Minor keys

We have seen the shape for minor keys. It was normal at Bach’s time to allow also the melodic versions of the scale (in C minor, G, A, B, C ascending, C, B♭, A♭ descending), and this forms an extended shape for minor keys.

The algorithm takes in successive notes, until a unique key is determined as suggested. The subjects of the fugues stick to a key, so this is a very special situation, where a melody line usually establishes a key quickly.
Resolving ambiguity

In the metrical analysis, we saw that stresses outside the metrical grid do not appear until the grid is established. For key, similarly, pitches outside the basic scale do appear after the key is established, but usually not before that.

Here is an example:

The key is uniquely determined after the first 7 notes (end of first bar). Notice that the later notes notated C♮ and B♯ are different notations for the same pitch. The ideas so far give a the basis of an analysis of how the notes outside the pitches of the key relate to the notes in the key.
Outside pitches

The part of pitch-space identified by the first 7 notes (5 pitches) is this:

F  C#  G#  D#  A#  E#  B#
D  A  E  B  F#  C#  G#
B♭  F  C  G  D  A  E

For other pitches, look for a position in the grid closest to the pitches identified by the key – this suggests the implicit role for E and C#. 
Outside pitches ctd

One more rule is used where there is a sequence of (one or more) semitone intervals in the melody: the first and last pairs of intervals should be in some order between a note in the established scale, and a note found via the combination of one place right and one place up (ie perfect fifth and major third).

In the example, we only have isolated semitone intervals; this thus relates C♮ to B and B♯ to C♯.
So we get the following for the first 17 notes (outside notes in blue, semitone pairs with yellow background).

\[
\begin{align*}
F & \quad C\# & \quad G\# & \quad D\# & \quad A\# & \quad E\# & \quad B\# \\
D & \quad A & \quad E & \quad B & \quad F\# & \quad C\# & \quad G\# \\
B♭ & \quad F & \quad C & \quad G & \quad D & \quad A & \quad E
\end{align*}
\]
Resolving Ambiguity ctd

The key determination algorithms is not enough for all the examples in the 48; there is a problem if the notes so far go from allowing several keys to allowing no keys when one note is added (eg Fugue 14 book 1, see article).

To choose between possible hypotheses when extra notes do not help, prefer if possible:

- first the key whose keynote is the first note of the fugue
- otherwise the key whose fifth note is the first note of the fugue.

This is enough to correctly identify the keys of all the fugues. It is very specific to the style in question, however.
Cognitive Aspects

As for rhythmic analysis, the harmonic algorithm aims to model the process by which a listener can come to understand the harmonic context in which the work unfolds.

Part of this is the incremental nature of the processing, which establishes the key as soon as it is unambiguous.

Other styles of music would make this a lot harder for a machine – because it is harder for the listener, and the music deliberately plays on harmonic ambiguity.
So far, this says nothing about recognition of key change (modulation), which is an important aspect of tonality in WTM. Once a key is established, notes outside the key can be felt as decoration (without changing the key), or as establishing a new key.

We can imagine keeping a window of recent pitch events, and when the number of notes outside the scale is too large, reanalysing the key. However, the setting up of key change is usually more obvious than this to the listener.
Comparison

These ideas are still relevant (though the work was done some time ago).
Comparison of some algorithms in this area that also aim to capture aspects of musical cognition are in Carol Krumhansl’s book “Cognitive Aspects of Musical Pitch”, Oxford University Press, 2001, ch 4; this chapter is on-line:

http://tinyurl.com/yfxqrza
Other interesting related problems:

- Analysis from audio
- Non-incremental analysis
  given a whole piece (say Bach Prelude), estimate the overall key, or key areas.
  Here a machine can do arbitrary comparisons between chunks that are far apart, and in any order.
  This doesn’t claim to model human cognition, but can be very effective.
Harmonic Progressions

In the context of a particular key, harmony in WTM is organised around a notion of **harmonic progressions**, which correspond to chosen sets of notes from the given key.

For the major keys, the starting point is to consider the chords based on the notes of the major scale by picking 3 notes (a triad), numbered

\[ n, n + 2, n + 4 \]

in the scale.

Here and in many other places we consider that notes an octave apart are considered to be equivalent for harmonic purposes. These are notes given the same **name** (A, B, C, \ldots), and this corresponds to doubling the underlying main frequency of the pitch.
Major scale chords

Then the chords in a particular key can be numbered in Roman form, where

- I is the chord on the keynote (the tonic, the base of the scale);
- ii is the (minor) chord on the second note of the scale;
- iii is the (minor) chord on the third note of the scale;
- and so on (lower case Roman for minor chord)

Many simple songs make use of just a small number of these harmonies (IV – V – I).

In using these harmonies, there are many possible ways of playing the notes of the chord (which octave, which note is the lowest, simultaneously or successively).
Example

The well-known Canon by Pachelbel illustrates the situation.

Here a single progression is repeated many times, with different melody notes associated. The original version left freedom to one player to play the harmonies associated with the bass line as they felt appropriate. Thus the score does not say which exact notes to play — compare a guitar accompaniment with similar indication. The progression, in D major, is:

I V vi iii IV I IV V I . . .

The phrase returns to the start, so I is both initial and final harmony.
The ending of a phrase usually has a special role for the harmony.
Canon as it starts

violin

cello
Melody with harmony

Notice here:

- The melody notes relate strongly to the harmony:
  - In the initial slower section, each pitch of the melody is one of the pitches of the harmony;
  - In the faster section, the pitches are either in the harmony, or close in pitch to a harmony pitch, while remaining in the key.

These are features of the pitch organisation of WTM that we would like to be able to:

- analyse, and
- use to guide generation of music.
Summary

- WTM keys as corresponding to related scales with tonal centres;
- tuning systems;
- 2-dimensional array of pitches reflecting harmonic relations;
- algorithm for local key determination;
- basics of harmonic progression.