



Music Informatics

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- ▶ Different harmonic organisation
- ▶ Pitch classes and their transformations
- ▶ Computers and Composition?

So far we have mostly considered harmony from WTM, where there are standard notions of **key**, **cadence** and so on.

We saw that paradigmatic analysis can function without building in assumptions that are present in Lerdahl and Jackendoff's GTTM, for example.

There are many other ways in which pitch-space can be organised; today mostly look at the atonal or serial music which abandoned notions of key at the start of the last century, and then looked for other ways to organise harmony.

Some of the ways of describing harmonies here were developed in 1960s influenced by the early use of computers.

Allen Forte's book "The Structure of Atonal Music" gives a general theory that analyses this sort of music using the notion of **pitch-class sets** (introduced earlier by Babbitt).

There is an on-line account of the basic ideas, with an applet illustrating some of the operations, due to Jay Tomlin, at

<http://www.jaytomlin.com/music/settheory/help.html>

and a useful summary of the theory from Paul Nelson:

<http://composertools.com/Theory/PCSets>

We are working here in a situation of equal temperament, where there is no pitch distinction between $c\sharp$ and $d\flat$. In a pitch-class set, pitches are taken modulo the octave, so octave displacements are ignored. The following example is due to Forte.

Compare ending of piece by Schönberg (the final chord below, from The Book of the Hanging Gardens, Op 15, no 1) to the start of piece by Webern (6 Pieces for Orchestra Op 6, No 3):



The image displays two musical excerpts side-by-side. The left excerpt is from Arnold Schönberg's 'The Book of the Hanging Gardens, Op. 15, No. 1', showing a complex, atonal chord structure in treble clef with a forte (*sf*) dynamic, and a bass line with a half note and a quarter note. The right excerpt is from Anton Webern's '6 Pieces for Orchestra, Op. 6, No. 3', showing a similar atonal chord structure in treble clef with a pianissimo (*pp*) dynamic, and a bass line with a half note and a quarter note.

Now move by notes to fit pitches in the smallest possible range.



So regard these chords as (transposed versions of) the **same** pitch-class set.

There is some controversy as to whether this sort of relation can be consciously heard by listeners; but this does indicate some sort of similarity; and we can say more about this in this case.

Since we don't care about transpositions, or the order of notes, then there is a simple way to describe pitch-class sets.

- ▶ Number the pitches from 0 to 11 (0 for c natural, 11 for b natural).
- ▶ Just give set of integers – traditionally here, write [0,2,5].

It turns out we can get a unique representation by finding fitting notes into smallest range by doing octave transpositions, and transposing so that the bottom note is C (=0); however

- ▶ if this does not uniquely specify the representation:
choose lowest difference between first two entries
- ▶ if this does not uniquely specify the representation:
choose lowest difference between first and third entries
- ▶ ...

The unique resulting representation is called the **normal form**.
Some operations relate sets together, in a way similar to operations in music using strict counterpoint.

- ▶ **Transposition**: as in the example above, take intervals in the same direction:

$[C, E, G] \Rightarrow [F, A, C]$; using shared pitch classes get:

$$[0, 4, 7] \Rightarrow [5, 9, 12]$$

and these have the same normal form.

Another relation between pitch classes is inversion:

- ▶ **Inversion**: take intervals in the opposite direction:
 $[C, E, G] \Rightarrow [C, A^b, F]$; after getting normal representation if the sets, get:

$$[0, 4, 7] \Rightarrow [0, 3, 7]$$

Two pitch sets are called **equivalent** iff they have the same normal form, or the normal form of one is the normal form of the inversion of the other.

The **prime form** of a set is its normal form, or the normal form of its inversion, whichever is smaller. (see Jay Tomlin's page).

Thus the major/minor distinction is abolished (in that the chords are equivalent).

Another relation is that between a pc set and its **complement**: the pitch-classes that are **not** in the given pitch class.

So [C,D,E \flat , F, G, A] \Rightarrow [C \sharp ,E,F \sharp ,G \sharp ,B \flat ,B] or

$$[0, 2, 3, 5, 7, 9] \Rightarrow [0, 2, 4, 6, 7, 9]$$

Can we hear this?

Probably not – but the literal case, where no transposition or inversion is involved, does have the effect of filling up the space of semi-tones. (The version with note names is this literal case, pc sets are not.)

There is also the relation between 2 pc sets, where one has takes only some of the pitch classes from the other – making it a **subset**. We know the literal version of this from classical harmony, where the major triad is a subset of the chord with the flattened 7th added.

Note that if we use unique representations, it might not be obvious which set is a subset of another, since transposition and inversion are allowed. eg,

$$[0, 1, 6] \subset [0, 2, 3, 8]$$

These ideas are influenced by the mathematical notion of set, as was Xenakis in his book “Formalized Music: Thought and Mathematics in Composition”.

In Forte's work, these ideas are part of a way of **analysing** music:
– look for relations between the pitches in groups of notes.

This takes us back to a familiar issue:

- ▶ segment the musical surface (into small segments here), and then compare pitch classes, or
- ▶ use pitch classes to recognise segmentation

When done by hand, there is a mixture of both processes.



A further idea is that a bunch of pitch class sets can be related together in a **complex**, e.g.

$K^*(T)$ the collection of the sets that are subsets of a given set T ;

$K(T)$ the collection of sets that are subsets **or** subsets of the complement of the given set T ;

$Kh(T)$ the collection of sets that are subsets **and** subsets of the complement of the given set T .

The score is below for this very short piano piece. There is an analysis by Forte (Forte, “The Structure of Atonal Music”, pp 97, 98)

The music has echoes of romantic harmony, but does not fit into WTM tradition for overall rhythmic or harmonic structure (Forte, “The Structure of Atonal Music”, pp 97, 98).

Still, it does not at all sound **random** to the listener.

Forte's analysis gives a way of understanding the relationships between parts of the musical material. It uses Forte's own terminology such as names for the unique pc set representations which are here attached to the score.

The details are not so important here; what is claimed is that this abstract representation relating pitches explains some of the ways in which this piece works as music (as far as pitch organisation is concerned). It acts as a replacement for classical notions of major/minor/diminished chords, their inversions, and classical harmonic progressions.



Opus 19, No. 6

Schoenberg

Sehr Langsam

pp *p* *pppp* *pp*

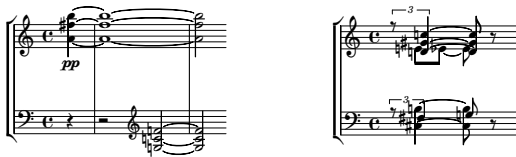
ppp *p* *ppp*

mit sehr zartem Ausdruck *genau im takt* *wie ein Hauch*

pppp

Hidden connections?

The pitch class set for the opening 6-note combined chord (repeated twice later) is also found twice in the second last bar (among the 9 pitches played)



The image displays two musical staves. The left staff shows a 6-note chord in treble clef with a piano (*pp*) dynamic marking. The right staff shows a 9-note passage in treble clef with a triplet of eighth notes in the second bar.

Using a computer to **find** and/or **verify** such analyses has benefits:

- ▶ It is easy to get things wrong working by hand, by claiming relations which are or, or (more likely) missing relations which hold.
- ▶ It forces the analyst to be precise about the relations involved, and what exactly is being looked for.

It is much easier to see how analysis can be implemented on the basis of this theory than say using GTTM.

paper on basic capabilities:

[http://music.columbia.edu/~akira/JDubiel/
PaperOnJDubielICMC09.pdf](http://music.columbia.edu/~akira/JDubiel/PaperOnJDubielICMC09.pdf)

and applied to George Perle's very similar ideas, from Computer Music Journal

[http://www.mitpressjournals.org/doi/pdf/10.1162/comj.2006.
30.3.53](http://www.mitpressjournals.org/doi/pdf/10.1162/comj.2006.30.3.53)

In the latter case, as in the L&J approach to grouping, there is still a large amount of ambiguity, and a good tool will allow an analyst to steer the way through to a preferred reading of segmentation on the basis of 12-tone pitch relationships.

Look at end of CMJ paper:

We have demonstrated in the present work a program that can compute useful information given a sequence of chord segments, performing in seconds what could take weeks or months for a human analyst. The natural question to ask is how much more can be automated, and what tasks absolutely require the expertise of a human analyst. . . .

When working with segments of fixed sizes (three to six in the current work), the number of possible arrangements is small enough to deal with. However, if an algorithm is given an entire composition, there are so many possible ways to segment it that it would not be possible to do an exhaustive analysis, even with a computer. We must find some heuristics . . .

These ideas have also been an explicit part of the compositional process for some composers. Can the computer help here too? From CMJ article:

the composer may enter input from a variety of perspectives, including potential pitch class segments, or desired interval cycles . . . and the application will provide as output a wealth of information for the composers consideration. In short, the program can supply the composer with a comprehensive set of pre-compositional material, based on the composers specifications. As such, then, the T3RevEng program is a practical, robust tool for both analysts and composers working within the parameters of twelve-tone tonality.

- ▶ Pitch classes as an abstract representation of pitch organisation (particularly in atonal music)
- ▶ Computer assistance using these abstractions for analysis and composition.