Music Informatics

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Today

- WTM pitch and key
- tuning systems
- a basic key analysis algorithm

WTM pitch organisation

Just as most WTM is organised around a regular metrical hierarchy, the pitches are typically organised around a standard set of (relative) pitches, where:

- the interval of the octave plays a key role:
  notes are named (like g♯) independently of the octave, voices singing an octave apart are singing "in unison".
- the intermediate notes are spaced roughly equally between the octave, as on piano keyboard or guitar frets.

This doesn't stop other pitches appearing, for expressive or other purposes. But the claim is that pitches are related to such an underlying framework.

Key

WTM is organised around a system of **keys** (like A major, B♭ minor).
A key is associated with a tonal centre (A) and a scale selected from the 12 semi-tones, in the major or minor pattern.
Notice that the names chosen for the notes on a keyboard are different depending on the key (g♯,a♭), and a system to turn a midi file into a conventional score has to work out the right spelling for the note. This depends on determining the key from a set of notes.

Tuning

There are different ways in which the semitone pitches have been related to each other historically, going back to Pythagoras. There was a big change at the time of Bach which allowed keyboard instruments to play in “remote” keys (like F$\sharp$ major) and still sound “in tune”. For an overview of these different tunings, see eg

http://tinyurl.com/yhtxs2k

The perfect fifth (eg C – G) is realised as the interval between the second and third harmonics (of the note an octave below the C here); physically these notes have frequencies in the ratio 2:3. Singers will aim for this tuning. In the Pythagorean tuning, major fifths are tuned exactly up and down from the tonic.

Pythagorean vs mean vs equal tuning

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Pythagorean
Total Intervals:1       9:8     81:64     4:3       3:2     27:16  243:128     2
Note:           C        D        E        F         G        A        B        C
local Intervals:   9:8      9:8    256:243   9:8       9:8      9:8   256:243
Cents:             204      204      90      204       204      204      90
```

The usual tuning before 1700 was just temperament (or mean tone), where major thirds are tuned exactly:

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Mean/just
Total Intervals:1                5:4                                           2
Note:           C        D        E        F         G        A        B        C
Cents:             193      193    117       193       193      193    117
```

The equal temperament scale makes every semitone the same size. So tone/semitone have size 200/100 cents.

It is not always easy to hear the difference between tunings; it is worth listening several times here.
Some examples of just tuning with equal temperament are at

> *http://www.wmich.edu/mus-theo/groven/compare.html*

Note that this has been set up so that just temperament is used even for a piece in C♯ major, by basing the tuning on that key (unlike in historic keyboard instruments).

Local key-finding in equal temperament

Problem: given a sequence of pitches in equal temperament (just as a semitone position on a semitone scale), estimate

- whether the melody is major or minor
- what the tonal centre is (keynote).

Longuet-Higgins developed a geometrical representation of semitone pitches in a 2-dimensional array that allows for multiple occurrences of the same (in equal temperament) pitches, depending on their role in different keys.
The algorithm tries to match a given set of pitches to possible occurrences on the array, so as to have the occurrences as close together as possible. The shape of the occurrences then suggests major or minor, and the keynote.

The basis for the two dimensional array is

- intervals horizontally are perfect fifths (7 semitones)
- intervals vertically are major thirds (4 semitones)

For the C major scale, this gives

A	E	B	
F	C	G	D

and for (harmonic version of) C minor, get

		B	
F	C	G	D
	A♭	E♭	

The array

This pattern in the general array indicates major and minor scales, with the keynote at a distinguished position. Recall that the spelling of the note name depends on position in the array.

E	B	F♯	C♯	G♯	D♯	A♯	E♯	B♯
C	G	D	A	E	B	F♯	C♯	G♯
A♭	E♭	B♭	F	C	G	D	A	E
F♭	C♭	G♭	D♭	A♭	E♭	B♭	F	C

This array can be extended, eg above and below where double sharp/double flat spellings are found.

The aim of the algorithm, given a sequence of pitches, is to look for a mapping of pitches into the array that corresponds to the major or minor key shape. This can be done by looking at each pattern of pitches corresponding to a key, and seeing how many of the given notes can be mapped inside this pattern.

Look at the subject of fugue 5, book 2 in D major. Given a midi-like representation of the notes, how do we work out whether the last note is F♯ or G♭?

There are 6 distinct pitches; the key is ambiguous between D major and G major, but in either case the last note is F♯.

Example ctd

For G major:

F♯	C♯	G♯	D♯	A♯	E♯	B♯
D	A	E	B	F♯	C♯	G♯
B♭	F	C	G	D	A	E
G♭	D♭	A♭	E♭	B♭	F	C

This shape suggests a missing but implied C natural;
G♭ is far away from the active pitches.

Example ctd

There is a similar story for D major:

F♯	C♯	G♯	D♯	A♯	E♯	B♯
D	A	E	B	F♯	C♯	G♯
B♭	F	C	G	D	A	E
G♭	D♭	A♭	E♭	B♭	F	C

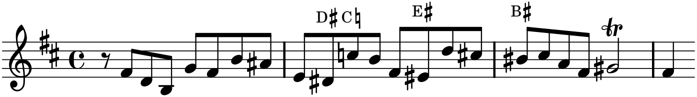
Now it is the C♯ that is suggested (and not D♭); it is a different occurrence of E also.

Minor keys

We have seen the shape for minor keys. It was normal at Bach's time to allow also the melodic versions of the scale (in C minor, G, A, B, C ascending, C, B♭, A♭ descending), and this forms an extended shape for minor keys.

The algorithm takes in successive notes, until a unique key is determined as suggested. The subjects of the fugues stick to a key, so this is a very special situation, where a melody line usually establishes a key quickly.

In the metrical analysis, we saw that stresses outside the metrical grid do not appear until the grid is established. For key, similarly, pitches outside the basic scale do appear after the key is established, but usually not before that.
Here is an example:



The key is uniquely determined after the first 7 notes (end of first bar). Notice that the later notes notated C♮ and B♯ are different notations for the same pitch. The ideas so far give a the basis of an analysis of how the notes outside the pitches of the key relate to the notes in the key.

Outside pitches

The part of pitch-space identified by the first 7 notes (5 pitches) is this:

F	C♯	G♯	D♯	A♯	E♯	B♯
D	A	E	B	F♯	C♯	G♯
B♭	F	C	G	D	A	E

For other pitches, look for a position in the grid closest to the pitches identified by the key – this determines where the D♯ and B♯ are located.

Outside pitches ctd

One more rule is used where there are (one or more) semitones in the melody: the first and last of any sequence of semitones should be made of the combination of one place right and one place up (ie perfect fifth and major third). This is what gives the difference between C♮ and B♯.

So we get the following for the first 17 notes (outside notes in blue, semitone pairs with yellow background).

F	C♯	G♯	D♯	A♯	E♯	B♯
D	A	E	B	F♯	C♯	G♯
B♭	F	C	G	D	A	E

Resolving Ambiguity ctd

The key determination algorithms is not enough for all the examples in the 48;
there is a problem if the notes so far go from allowing several keys to allowing no keys when one note is added (eg Fugue 14 book 1, see article).

To choose between possible hypotheses when extra notes do not help, prefer if possible:

- *first the key whose keynote is the first note of the fugue*
- *otherwise the key whose fifth note is the first note of the fugue.*

This is enough to correctly identify the keys of all the fugues.
It is very specific to the style in question, however.

Notating other notes

Harmonic analysis should also provide a way of notating the pitches for notes outside the notes of the scale once the key has been identified. Even in these short examples, we can see that Bach uses both B$\sharp$ and C$\natural$ in one phrase (fugue 24 book 1).
A rule to resolve this is that semitone intervals between notes should be notated as different notes in the scale, with one possibly sharpened or flattened from the normal note in the scale.
This rule works for these examples. Justifying this decision in terms of the theory of harmony of the time would take us out of the scope of the course ...

As for rhythmic analysis, the harmonic algorithm aims to model the process by which a listener can come to understand the harmonic context in which the work unfolds.

part of this is the incremental nature of the processing, which establishes the key as soon as it is unambiguous.

Other styles of music would make this a lot harder for a machine – because it is harder for the listener, and the music deliberately plays on harmonic ambiguity.

Modulation

So far, this says nothing about recognition of key change (modulation), which is an important aspect of tonality in WTM.
Once a key is established, notes outside the key can be felt as decoration (without changing the key), or as establishing a new key.
We can imagine keeping a window of recent pitch events, and when the number of notes outside the scale is too large, reanalysing the key.
However, the setting up of key change is usually more obvious than this to the listener.

Comparison

These ideas are still relevant (though the work was done some time ago).

Comparison of some algorithms in this area that also aim to capture aspects of musical cognition are in Carol Krumhansl's book "Cognitive Aspects of Musical Pitch", Oxford University Press, 2001, ch 4; this chapter is on-line:

http://tinyurl.com/yfxqrza

Other harmonic tasks

Other interesting related problems:

- Analysis from audio
- Non-incremental analysis
  given a whole piece (say Bach Prelude), estimate the overall key, or key areas.
  Here a machine can do arbitrary comparisons between chunks that are far apart, and in any order.
  This doesn't claim to model human cognition, but can be very effective.

In the context of a particular key, harmony in WTM is organised around a notion of harmonic progressions, which correspond to chosen sets of notes from the given key.
For the major keys, the starting point is to consider the chords based on the notes of the major scale by picking 3 notes (a triad), numbered

$$n, n+2, n+4$$

in the scale.

Here and in many other places we consider that notes an octave apart are considered to be equivalent for harmonic purposes. These are notes given the same name (A, B, C, ...), and this corresponds to doubling the underlying main frequency of the pitch.

Major scale chords

Then the chords in a particular key can be numbered in Roman form, where

- **I** is the chord on the keynote (the tonic, the base of the scale);
- **ii** is the (minor) chord on the second note of the scale;
- **iii** is the (minor) chord on the third note of the scale;
- and so on (lower case Roman for minor chord)

Many simple songs make use of just a small number of these harmonies (IV – V – I).

In using these harmonies, there are many possible ways of playing the notes of the chord (which octave, which note is the lowest, simultaneously or successively).

Example

The well-known Canon by Pachelbel illustrates the situation.

Here a single progression is repeated many times, with different melody notes associated. The original version left freedom to one player to play the harmonies associated with the bass line as they felt appropriate. Thus the score does not say which exact notes to play — compare a guitar accompaniment with similar indication. The progression, in D major, is:

I V vi iii IV I IV V I . . .

The phrase returns to the start, so **I** is both initial and final harmony.
The ending of a phrase usually has a special role for the harmony.

Canon as it starts

Melody with harmony

Notice here:

- The melody notes relate strongly to the harmony:
  - In the initial slower section, each pitch of the melody is one of the pitches of the harmony;
  - In the faster section, the pitches are either in the harmony, or close in pitch to a harmony pitch, while remaining in the key.

These are features of the pitch organisation of WTM that we would like to be able to:

- analyse, and
- use to guide generation of music.

Summary

- WTM keys as corresponding to realted scales with tonal centres;
- tuning systems;
- 2-dimensional array of pitches reflecting harmonic relations;
- algorithm for local key determination;
- basics of harmonic progression.