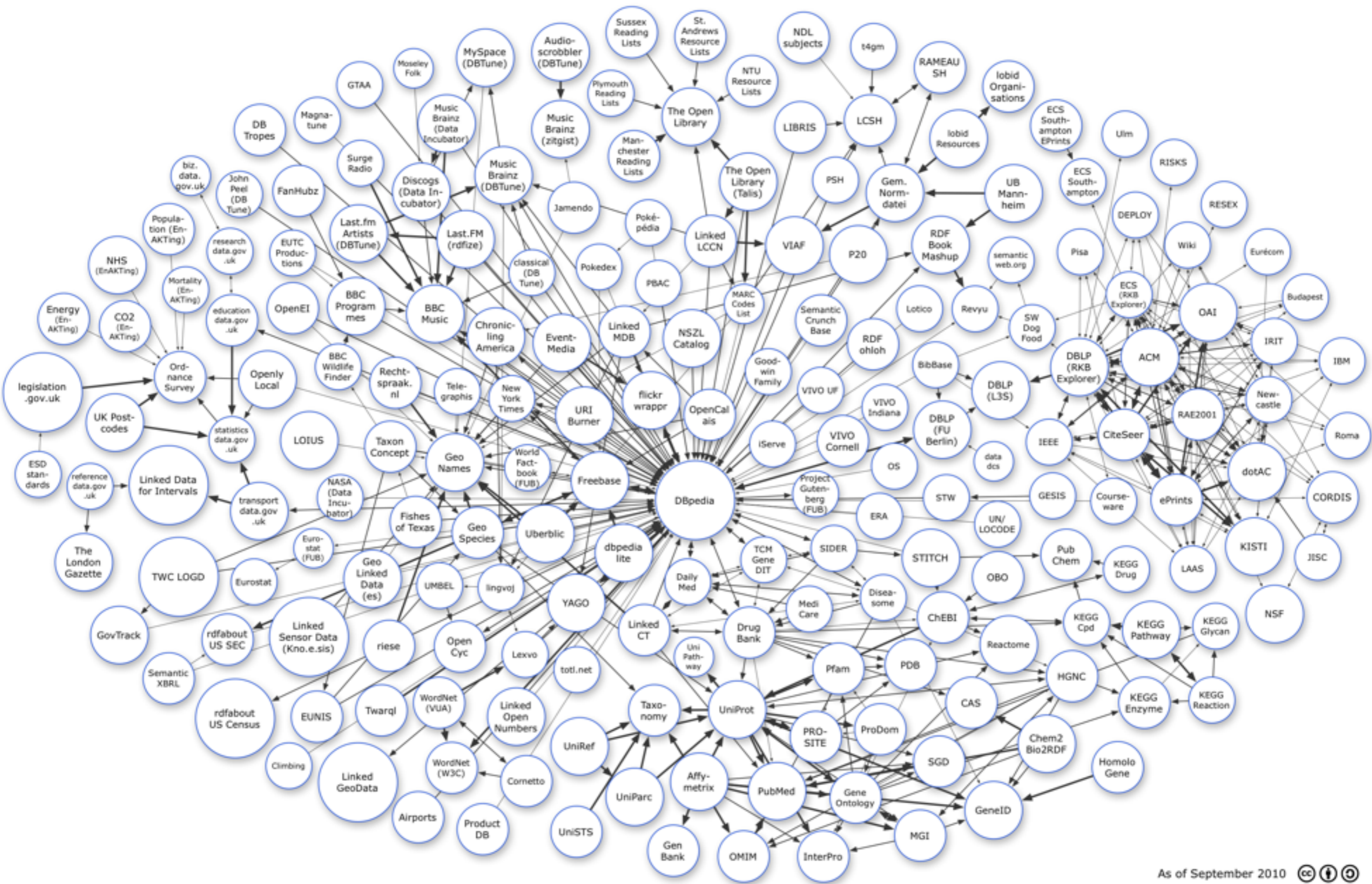


communities  
[ncw ch. 3]



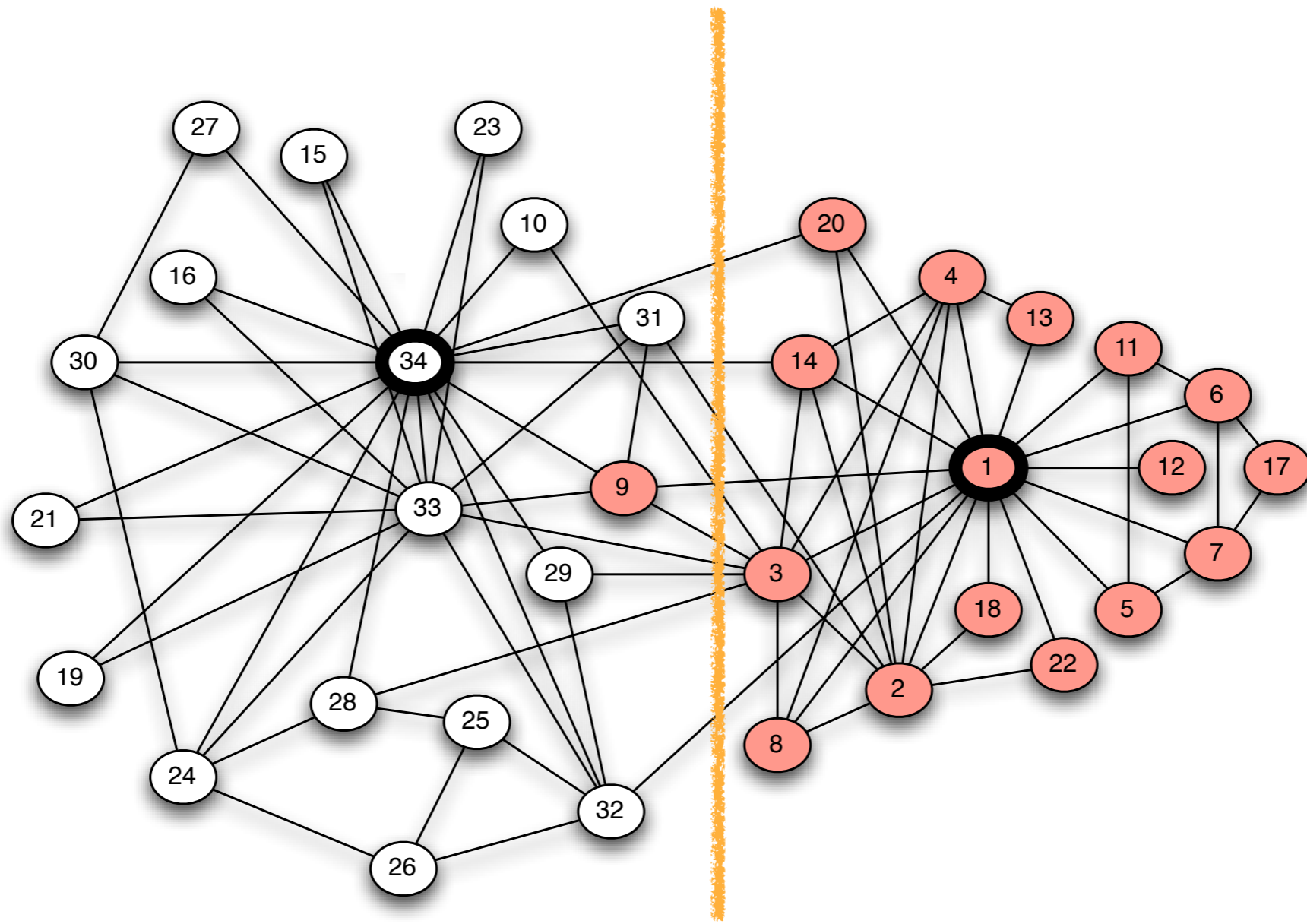
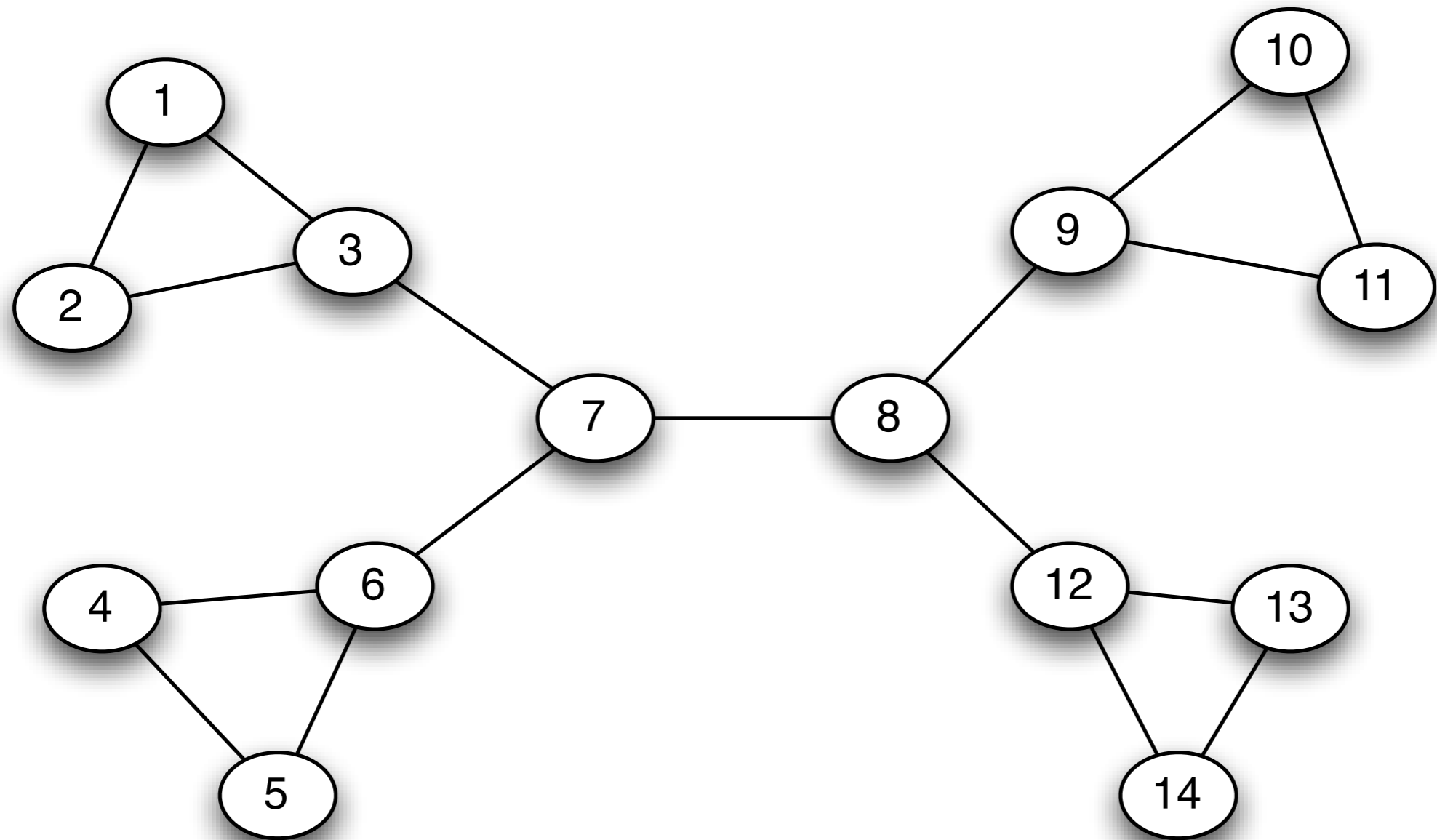
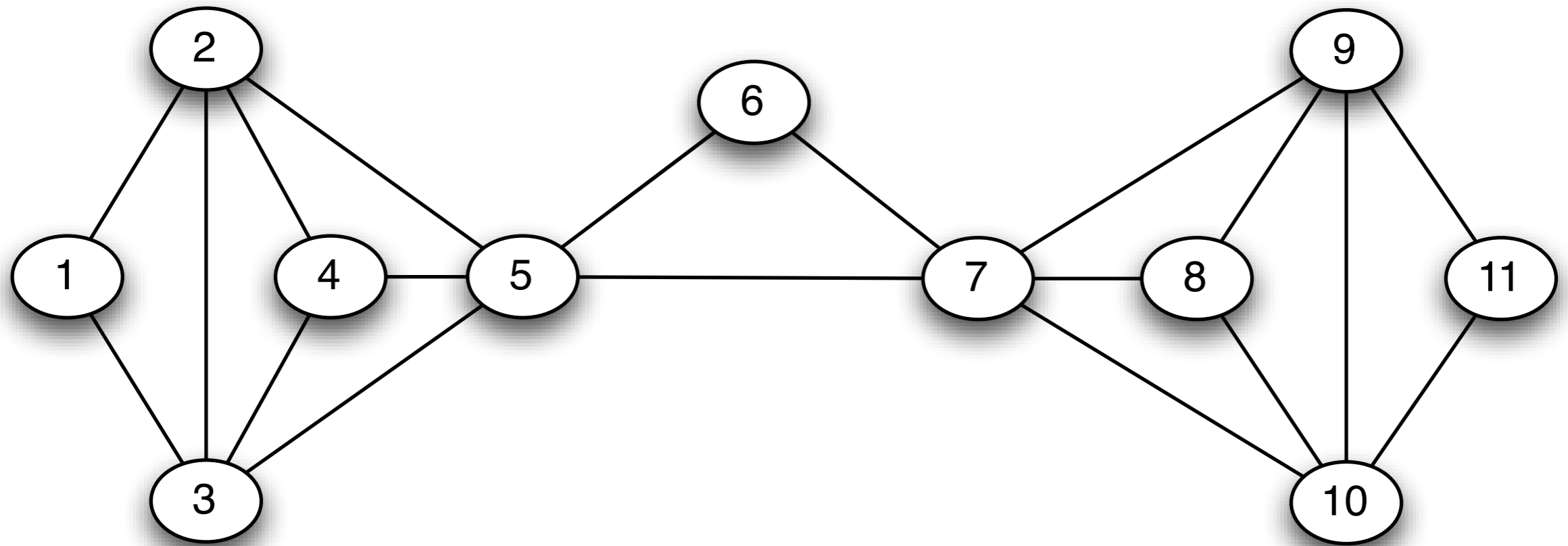


Figure 3.13: A karate club studied by Wayne Zachary [421] — a dispute during the course of the study caused it to split into two clubs. Could the boundaries of the two clubs be predicted from the network structure?

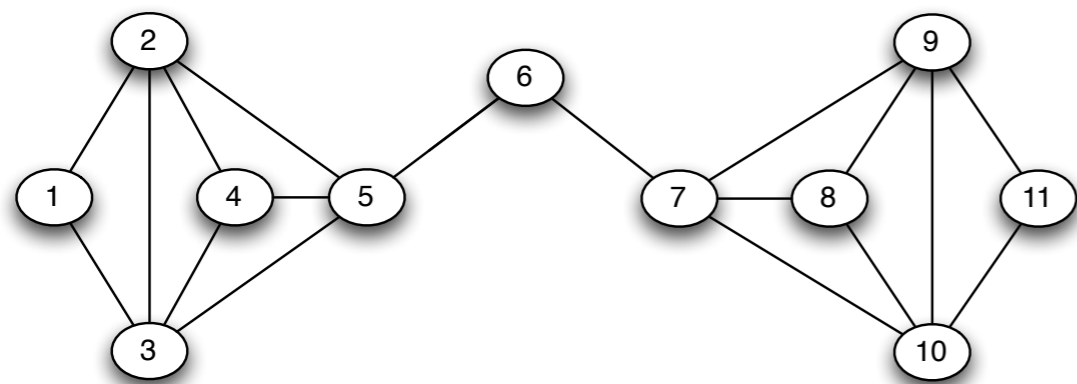
Define the *between-ness* of a node  $u$ , written  $\beta(u)$ , as the number of shortest paths of  $G$  containing  $u$  - that is to say:

$$\begin{aligned}\beta(u) &= \sum_{x \in V} \varphi_x(u) \\ \varphi_x(u) &= \sum_{y \in V} |\{\gamma \in sp(x, y) \mid u \in \gamma\}| \end{aligned}$$

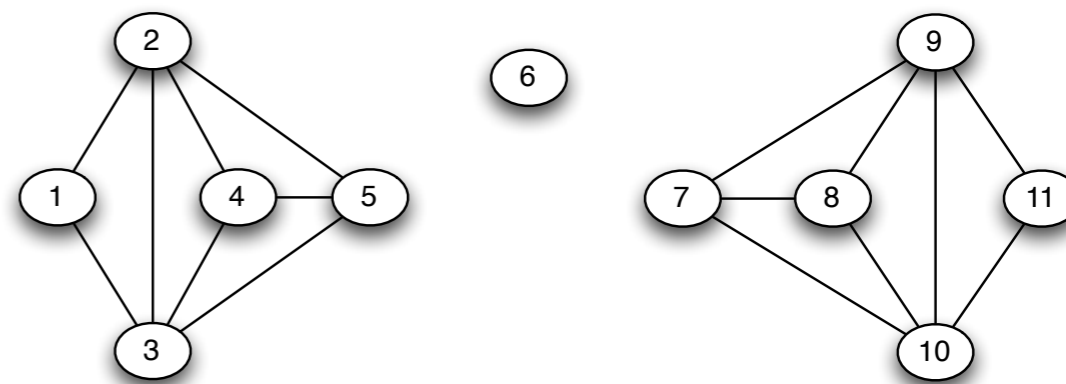




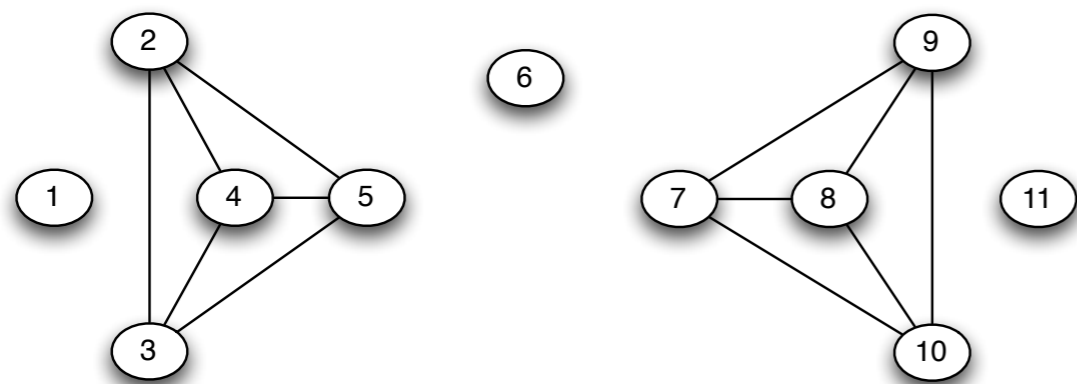




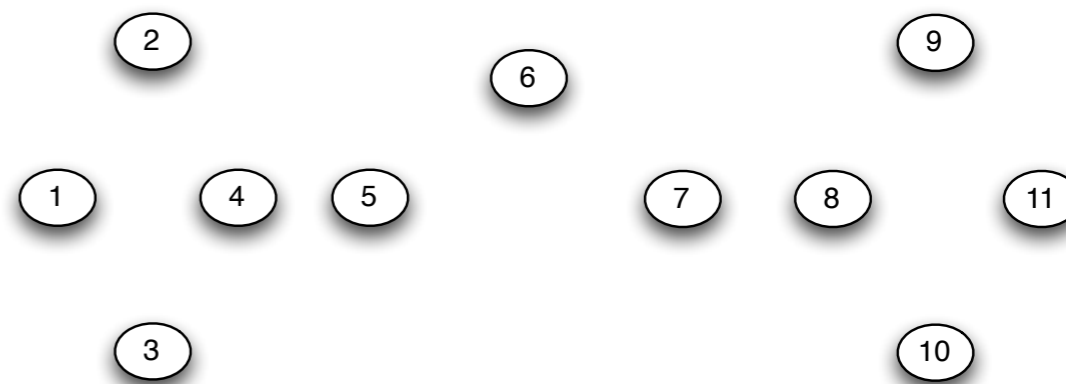
(a) *Step 1*



(b) *Step 2*

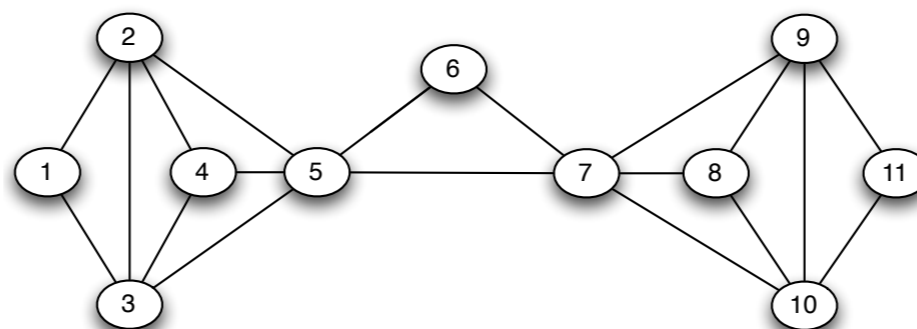


(c) *Step 3*



(d) *Step 4*

Figure 3.17: The steps of the Girvan-Newman method on the network from Figure 3.15.



Write  $BF(x)$  for the ranked graph induced by a *breadth-first* search from  $x$  in  $G$ , where each node  $y$  in  $V$  is mapped to  $d(x, y) \in \mathbb{N}$  its distance to  $x$ . Write  $sp(z, y)$  for set of shortest paths from  $z$  to  $y$  in  $G$ .



(i) Show that neighbours in  $G$  are at most one rank remote in  $BF(x)$ .

Show that deleting all edges  $(y, z)$  such that  $d(x, y) = d(x, z)$  (edges of constant rank) in  $BF(x)$  obtains a directed acyclic graph  $BF^-(x)$ .

Since,  $d(x, y) + d(y, z) \leq d(x, z)$ , if  $d(y, z) = 1$ , then  $|d(x, y) - d(x, z)| \leq 1$ . Directedness and acyclicity follow from the fact that all paths are now strictly rank-increasing.

Write  $ri(z, y)$  for set of paths from  $z$  to  $y$  in  $BF(x)$  with a strictly increasing rank; equivalently, the set of directed paths from  $z$  to  $y$  in  $BF^-(x)$ .

Write:

- $y^-$  for the set of immediate parents of  $y$  in  $BF^-(x)$
- $z^+$  for the set of immediate successors of  $z$  in  $BF^-(x)$

We have  $y^- = \emptyset$  iff  $y$  is the root  $x$  of  $BF^-(x)$ ,  $z^+ = \emptyset$  iff  $z$  is a leaf in  $BF^-(x)$ .

(ii) Set  $\sigma(x, y) = |ri(x, y)|$ , and show that this is the number of shortest paths from  $x$  to  $y$ .

Show that  $\sigma(z, z) = 1$ , and, conversely,  $\sigma(z, y) > 0, \sigma(y, z) > 0 \Rightarrow z = y$ .

(iii) Show that  $\sigma(x, y) > 0$ , and  $\sigma(x, z) \leq \sigma(x, y)$  if  $z \in y^-$ .

(iv) Show that  $\sigma$  verifies:

$$\begin{aligned}\sigma(x, x) &= 1 \\ \sigma(x, y) &= \sum_{z \in y^-} \sigma(x, z) \quad \text{if } x \neq y\end{aligned}$$

(vi) Define the *between-ness* of a node  $u$ , written  $\beta(u)$ , as the number of shortest paths of  $G$  containing  $u$  - that is to say:

$$\begin{aligned}\beta(u) &= \sum_{x \in V} \varphi_x(u) \\ \varphi_x(u) &= \sum_{y \in V} |\{\gamma \in sp(x, y) \mid u \in \gamma\}| \end{aligned}$$

Show:

$$\varphi_x(u) = \sigma(x, u) + \sum_{v \in u^+} \varphi_x(v) \cdot \sigma(x, u) / \sigma(x, v)$$

For every  $x$ , build  $BF(x)$  and compute for each node  $y$  the number  $\sigma(x, y)$  using the top-down formula of subquestion (iv); then use the bottom-up formula just above, to compute  $\varphi_x(u)$ ; to obtain  $\beta(u)$ , sum over  $x$ ; the complexity is linear in the number of edges and linear in the number of nodes, so quadratic in  $G$ , hence cubic if one wants to pick up nodes of maximal betweenness.



is between-ness a good notion?

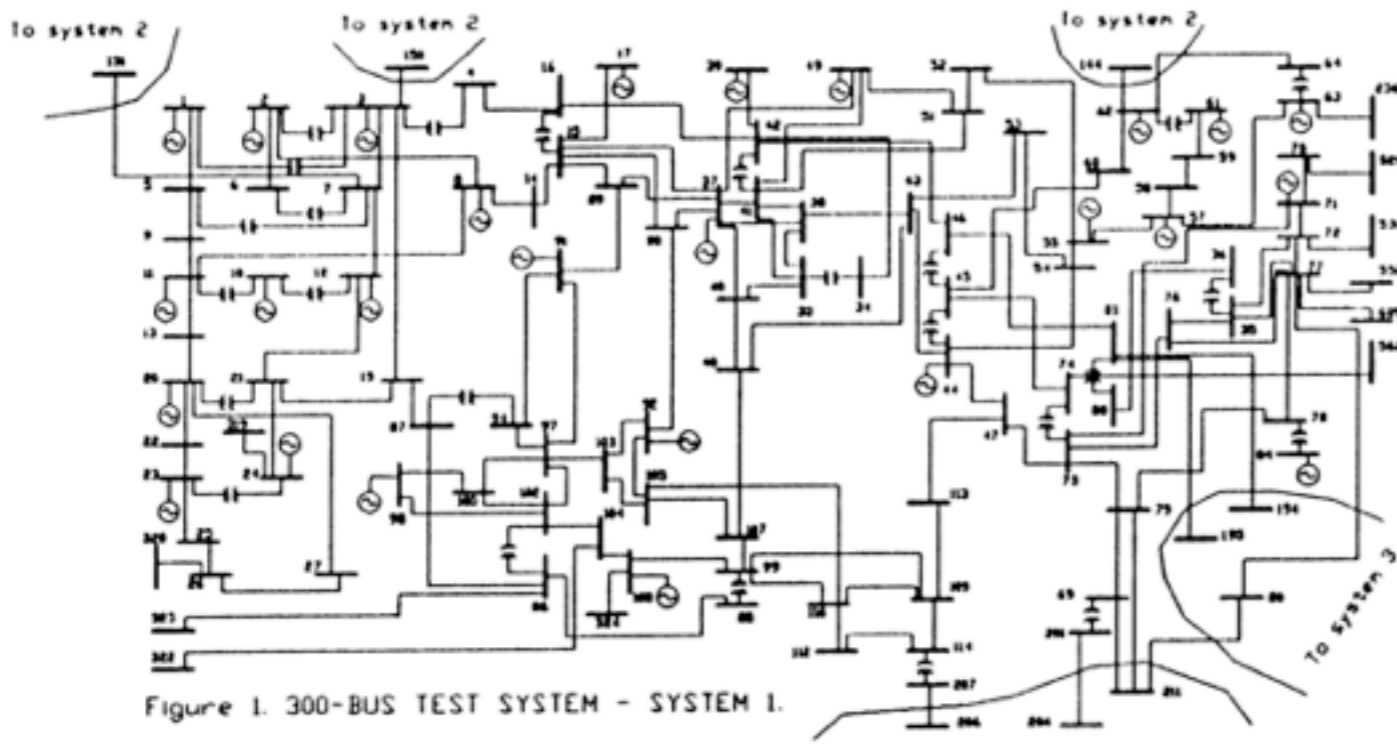


Figure 1. 300-BUS TEST SYSTEM - SYSTEM 1.

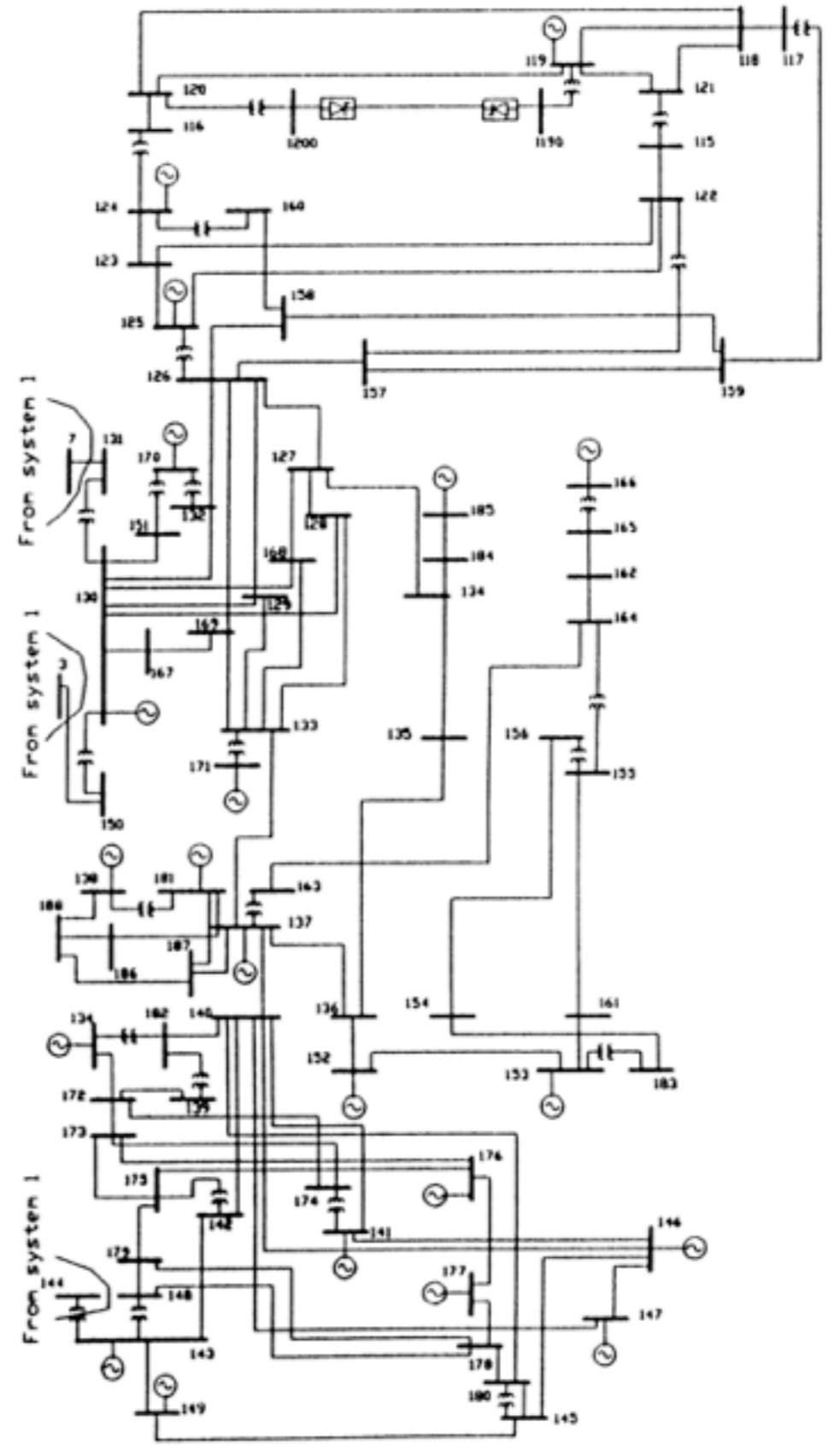


Figure 2. 300-BUS TEST SYSTEM - SYSTEM 2.

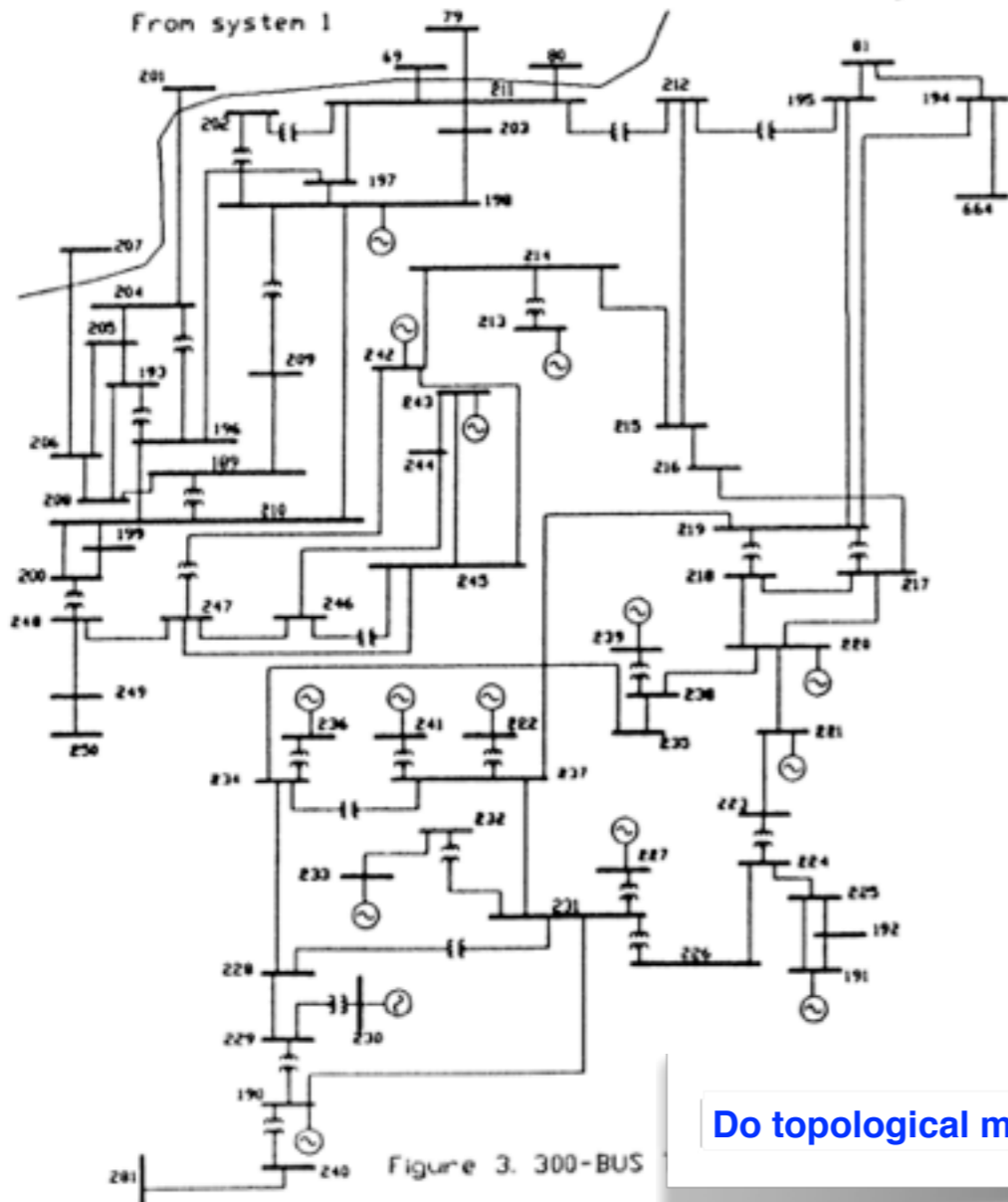


Figure 3. 300-BUS TEST SYSTEM - SYSTEM 3.

Do topological models provide good information about electricity infrastructure vulnerability?

## 3 measures of disruption/5 attack modes

Mean distance between nodes

Connectivity loss

$$C = 1 - \langle n(x, g) / n(g) \rangle$$

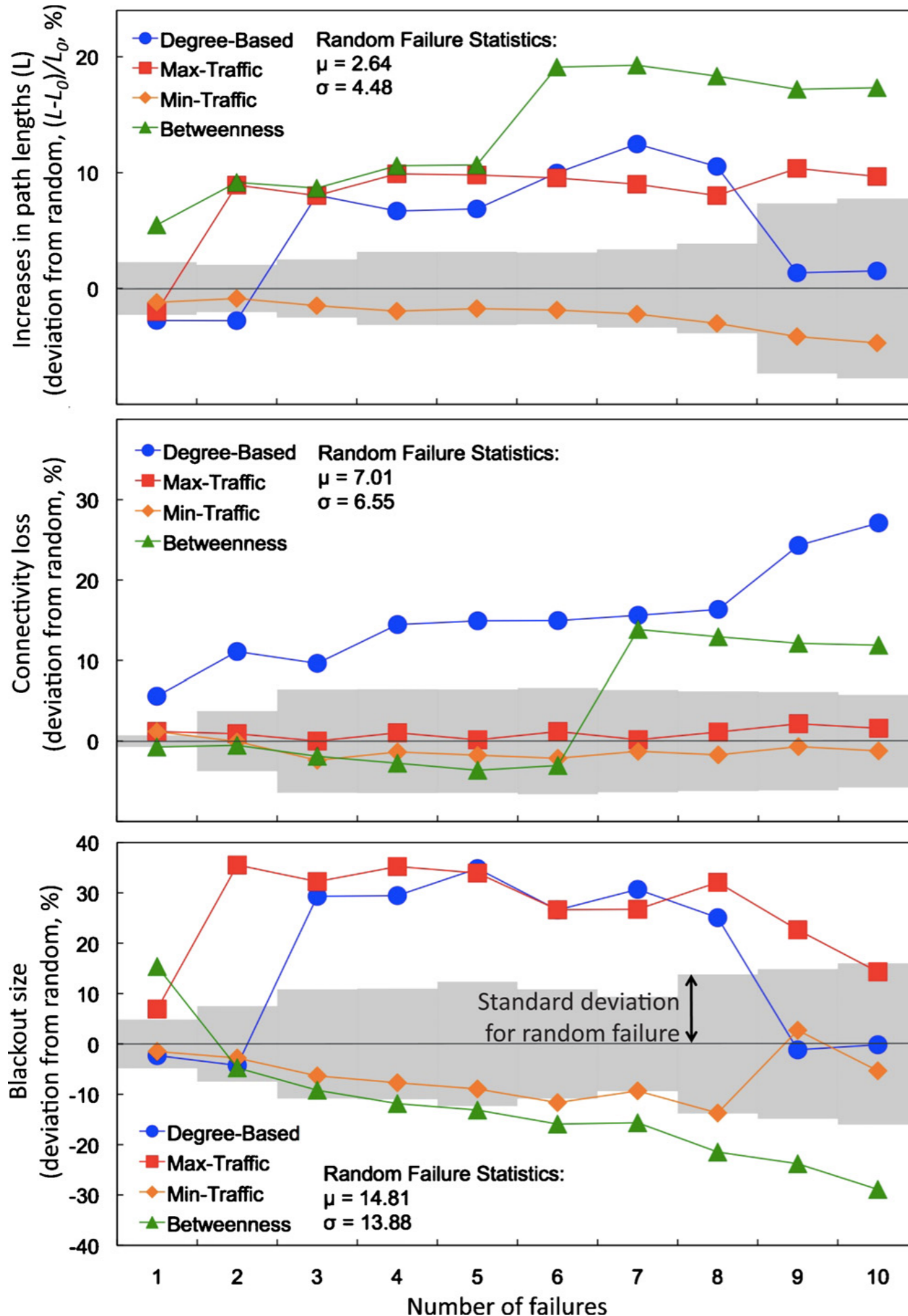
$n(g)$  = total number of generators

$n(x, g)$  = number of generators connected to  $x$

Blackout sizes

as calculated from a model of cascading failure in a power system

$$P_i = \sum_{j=1}^n (\theta_i - \theta_j) / X_{ij}$$



Simulated response of the IEEE 300 bus network to directed attacks

**top:** mean path length  
**middle:** connectivity loss  
**bottom:** size of blackout

random failures are averaged over 20 trials

differences between attack results and random failures

Shading  $\pm\sigma$  for random failures

**previously ...**

between-ness a measure of the flow through a node (or edge)

# today ...

triadic closure ...

strength of weak ties

architecture of social networks

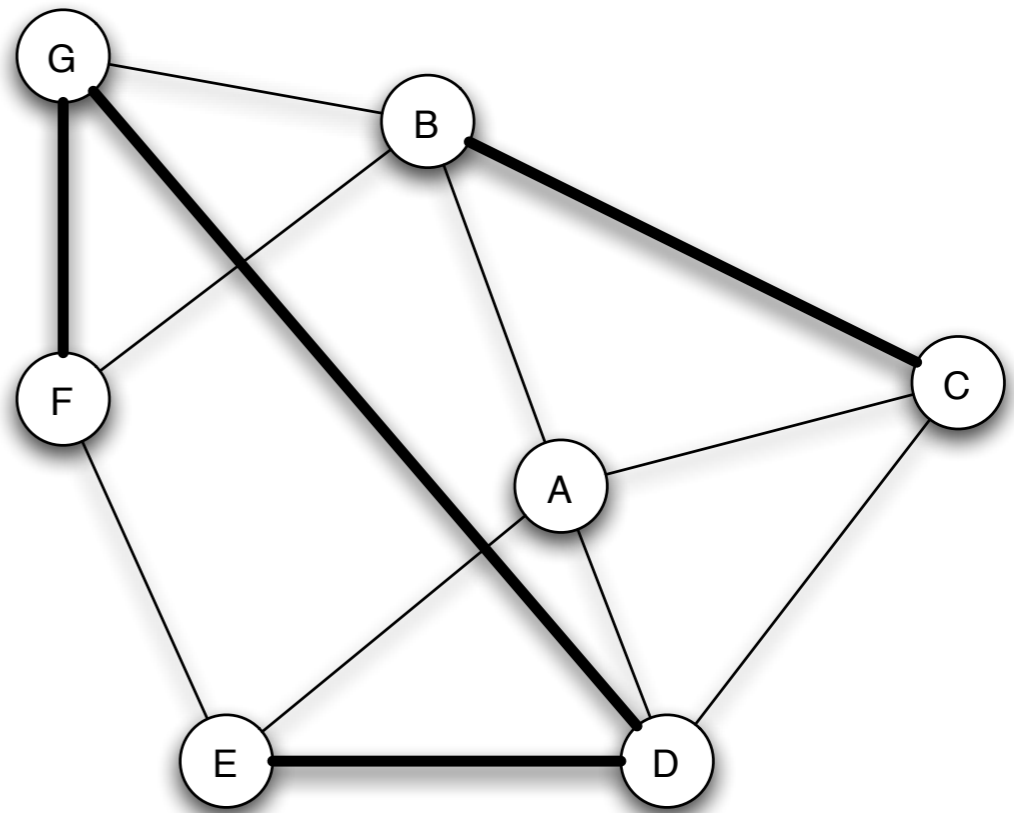
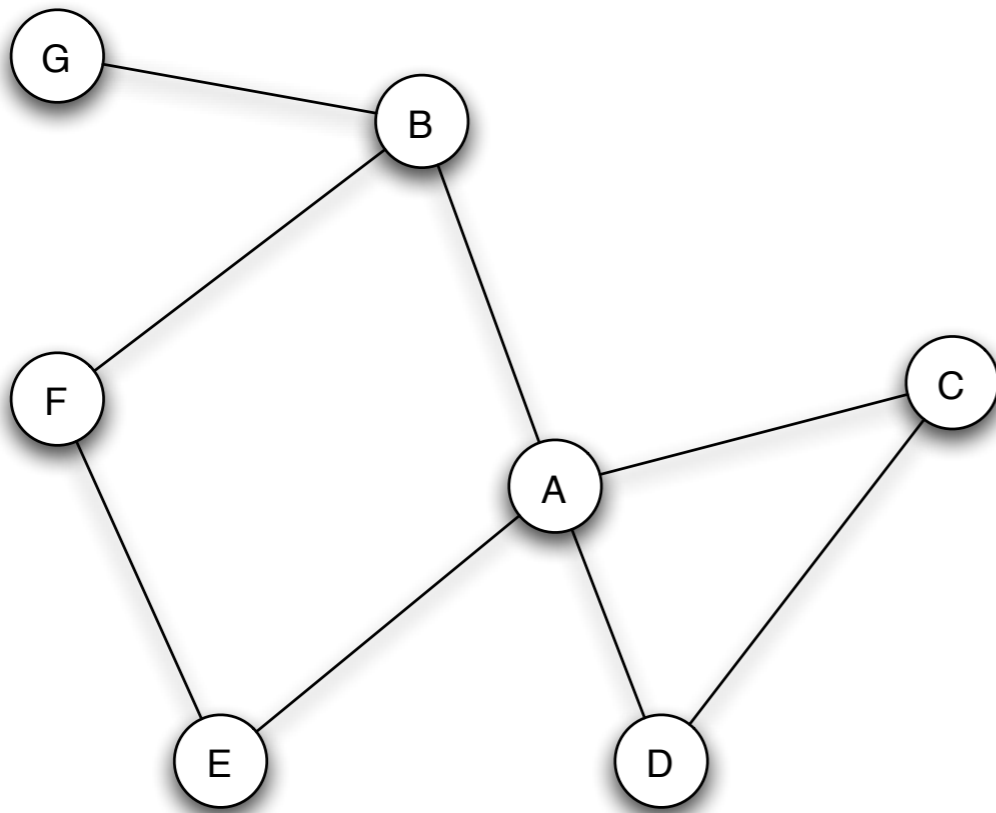
how a network evolves over time

interpreting between-ness

triadic closures ...

Clustering Coefficient(A) = fraction of A's friends who are friends

compute cc(A) before and after



opportunity

trusting

stress



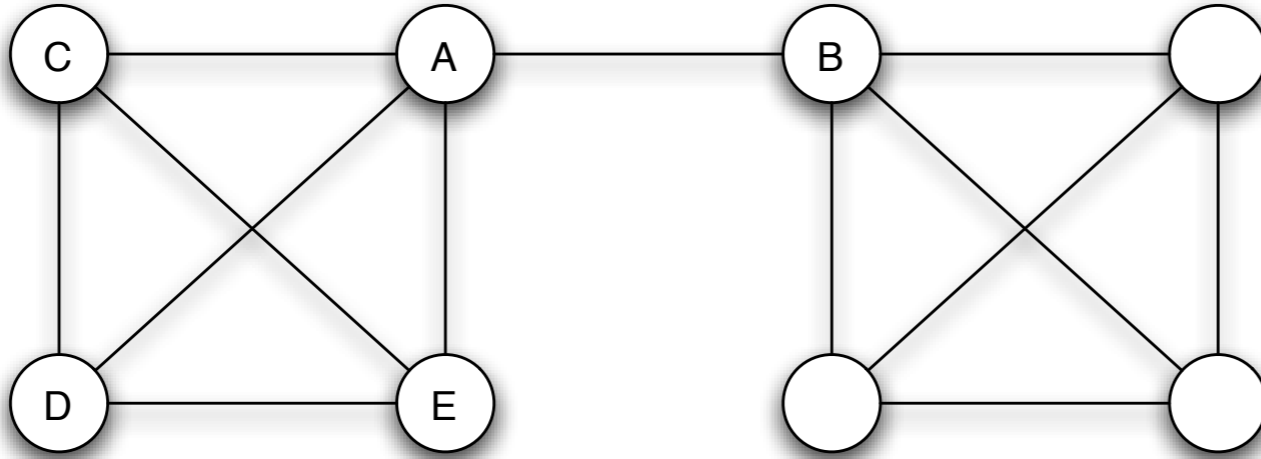
cc is genetically inherited

attach-and-introduce model

epistemo point: recreate the process behind a feature - see Miguel's lectures; also Panconesi's lecture on web compressibility

bridges

interpreting between-ness: where does info come from; different time scales and steady states?



local bridges

no friends in common

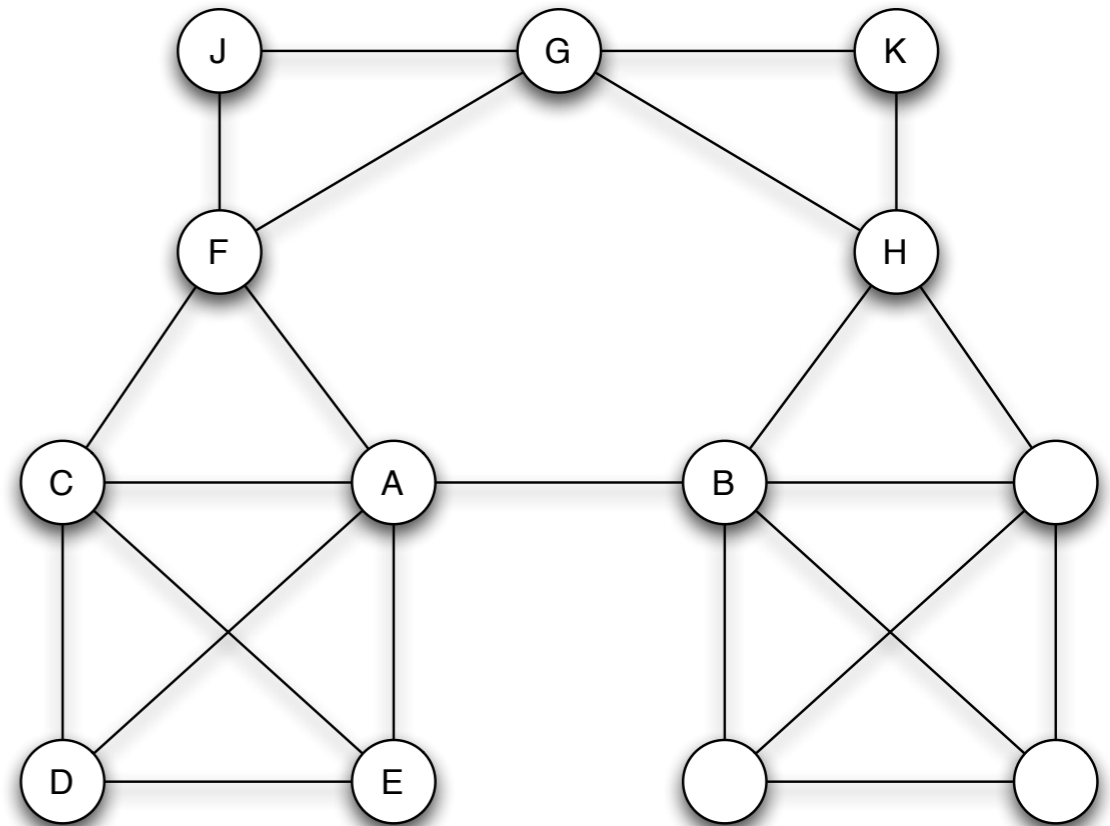
low cc

distance contraction, curvature?

compute the derivative of gossip upper bounds?

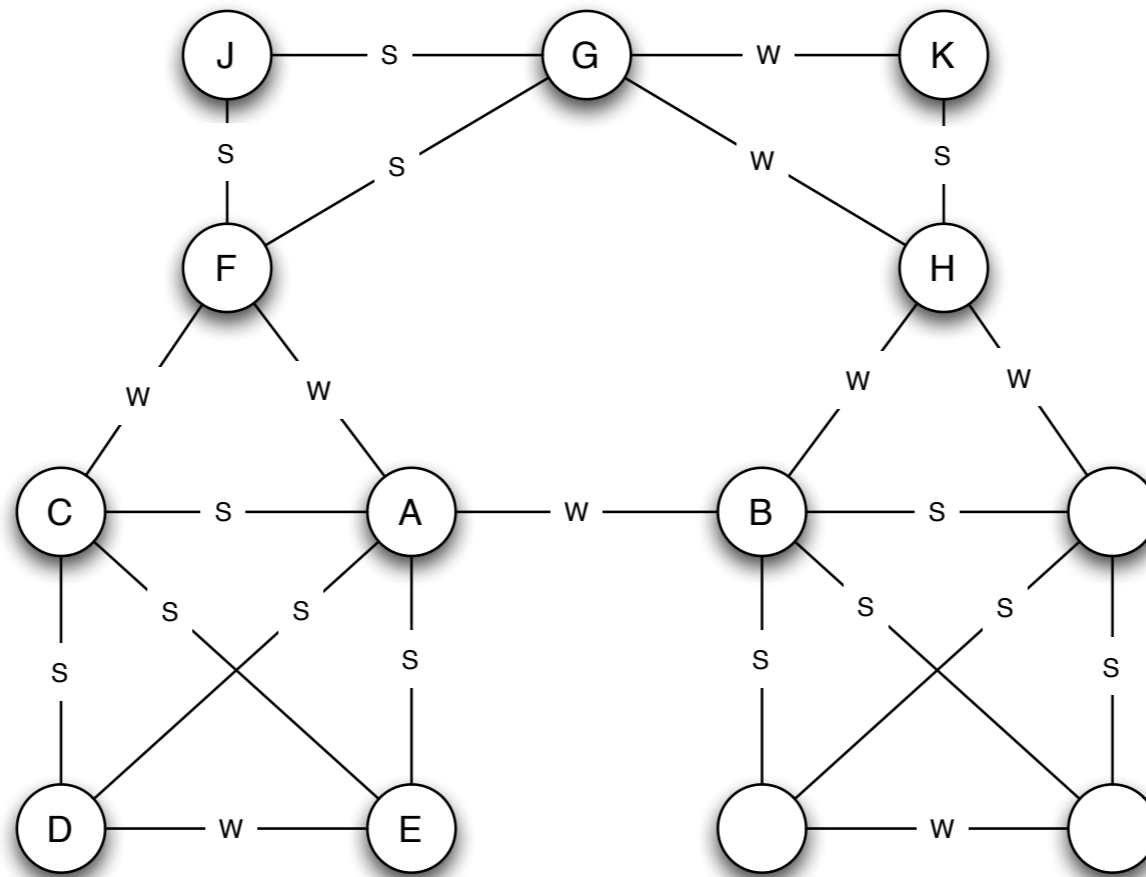
what is propagating?

philosophical stake: fine time-structure of information propagation



strong ties (the stronger links, corresponding to friends), and weak ties

strong triadic closedness



local bridge on A (2 s) is weak  $\leq$  STC

Strong Triadic Closure says the B-C edge must exist, but the definition of a local bridge says it cannot.

