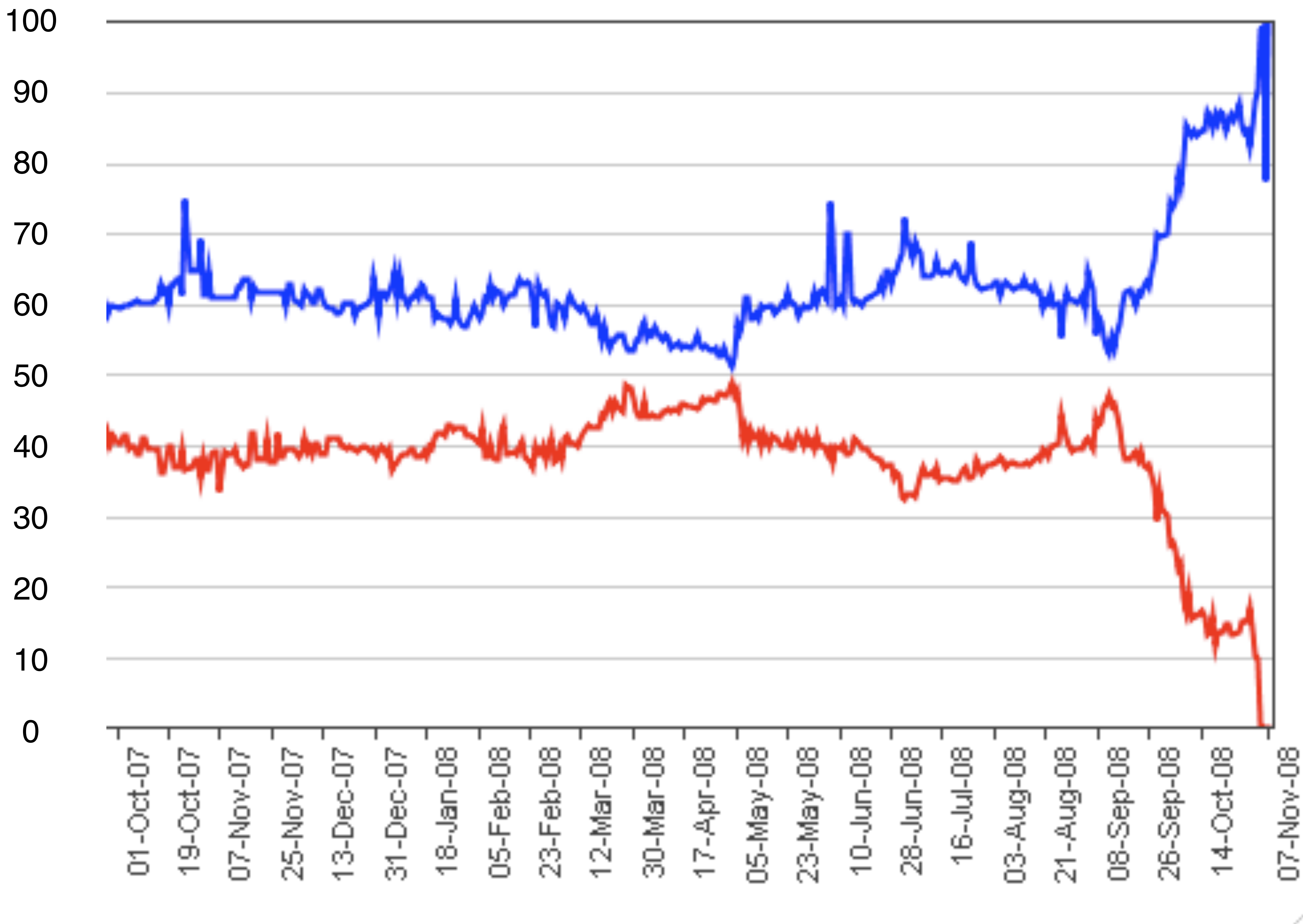
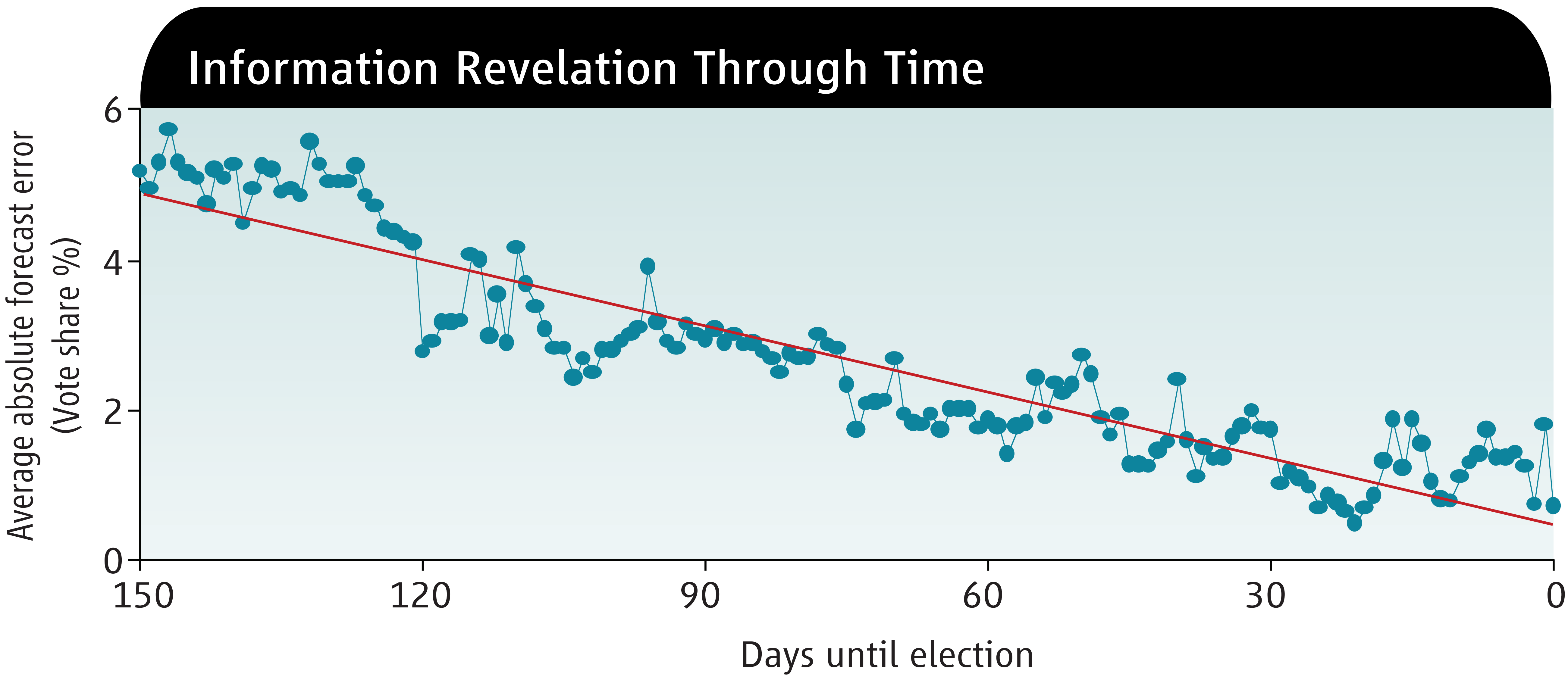
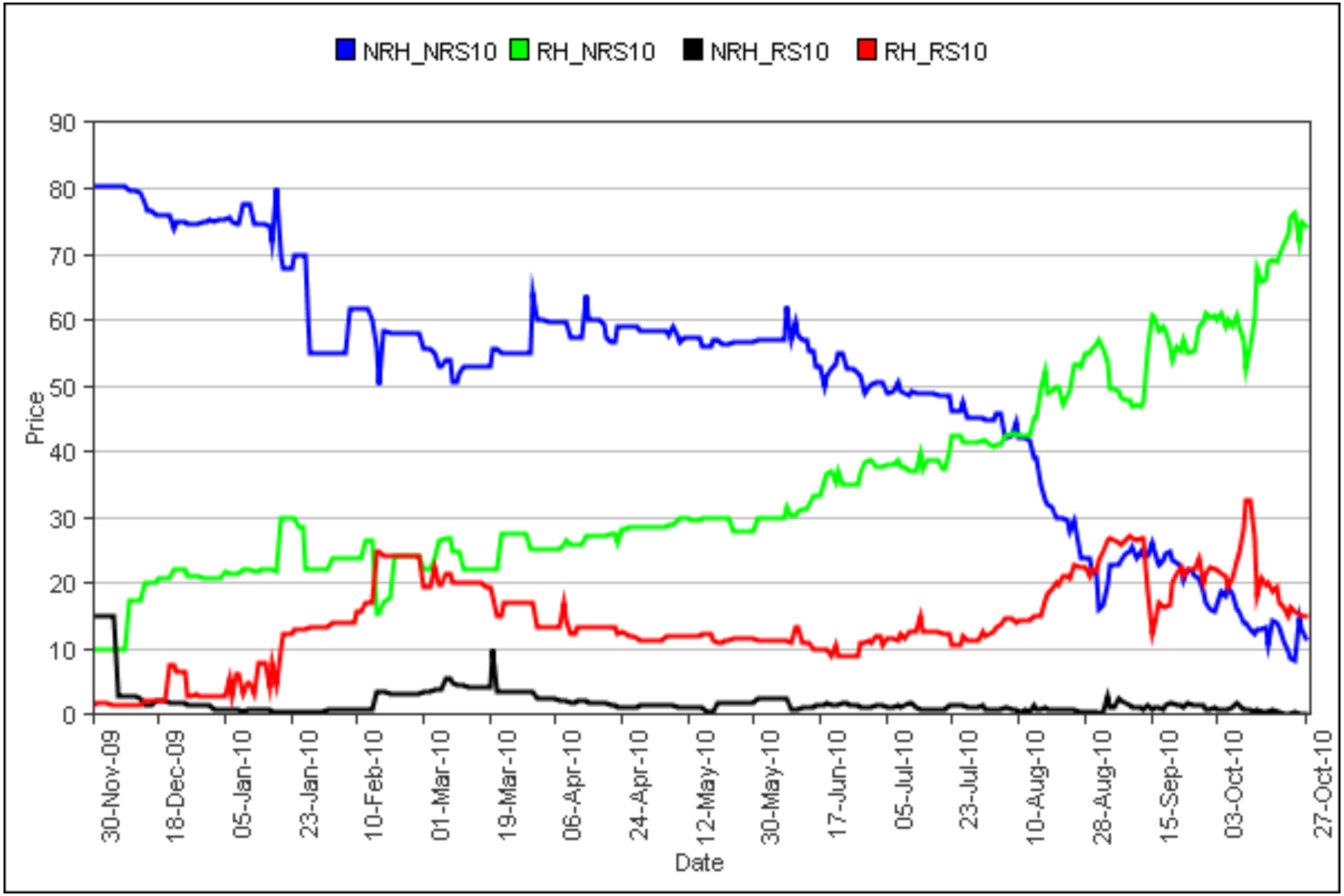
prediction markets
[ncw ch. 1]



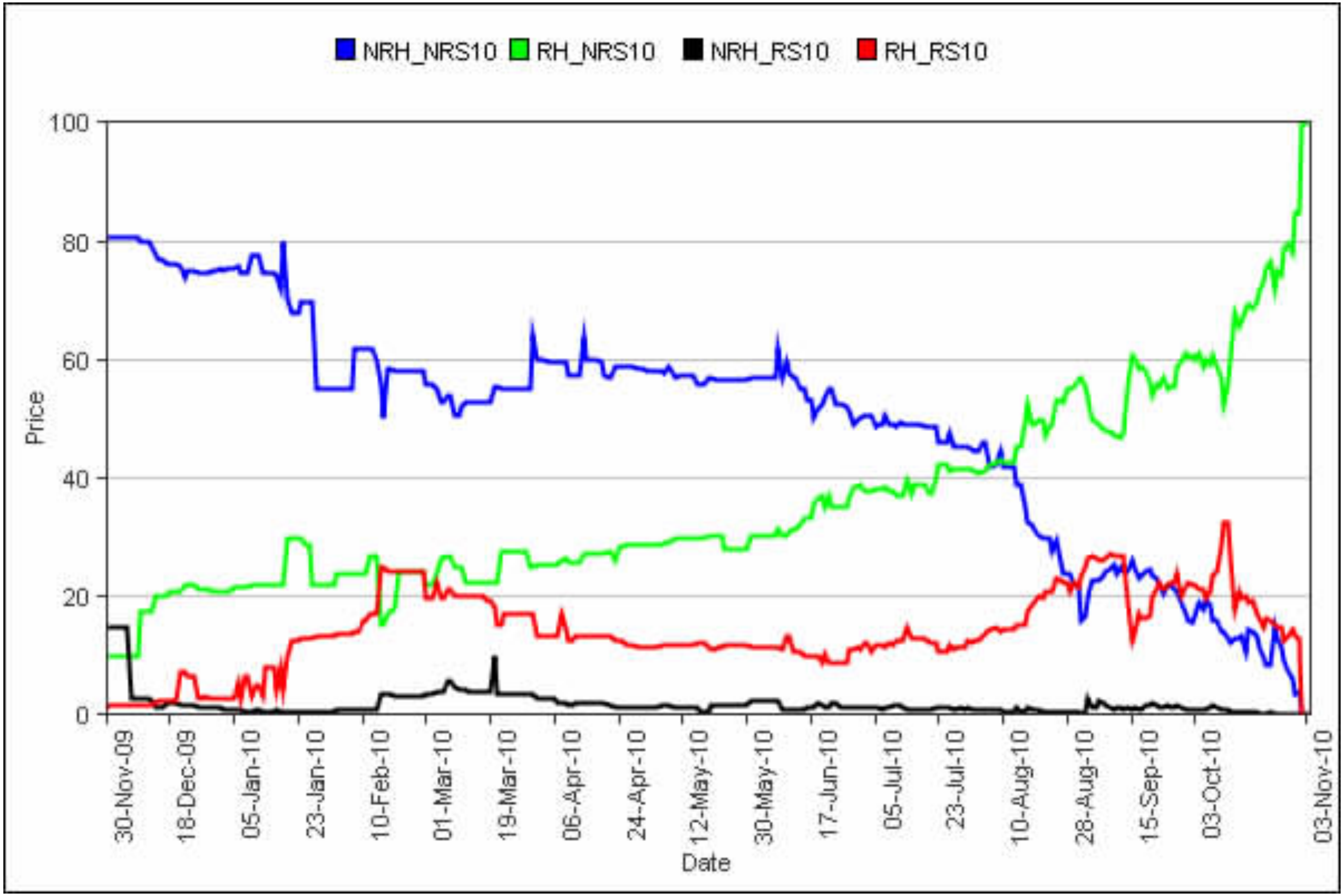
Arrow et al.
Science 320, 877-878(2008)



2010 US Congressional Control Market



liquidation time!



previously ...

agents

We consider a prediction market on a finite set K of outcomes with a finite set M of agents.

An agent $m \in M$ is a pair (β_m, ϕ_m) where:
- $\beta_m \in \mathbb{R}^+$ is the wealth of agent m
- $\phi_m^k(c) \in [0,1]$ is the fraction of β_m that agent m bets on outcome k
- and $c_k \in [0,1]$ is the price of the contract that wins \$1 if the outcome is k

$$0 \leq c_k \leq 1$$

```
total investment
```

total investment of agent m is $\beta_m \sum_k \phi_m^k(c)$

$$\sum_k \phi_m^k \leq 1$$

```
updated budget per agent
```

$$\beta'_m/\beta_m \quad = \quad 1 - \sum_k \phi_m^k(c) + c_y^{-1} \cdot \phi_m^y(c)$$

$$\sum_k \phi_m^k = 1?$$

market balance, fusion of beliefs

$$c_y \cdot \sum_{m,k} \beta_m \phi_m^k(c) \quad = \quad \sum_m \beta_m \phi_m^y(c)$$

$$\boxed{c_y = n_y(c)/n(c)}$$

$$\sum_k c_k = 1$$

total bet on k, total bet

$$\begin{aligned} n_k(c) &:= \sum_m \beta_m \phi_m^k(c) \\ n(c) &:= \sum_k n_k(c) \end{aligned}$$

reasonable conditions for existence and unicity of a market balancing price

assumptions about agents

We suppose that ϕ_m^k is such that:
- it only depends on c_k
- $\phi_m^k(0) > 0$
- $\phi_m^k(1) = 0$
- ϕ_m^k continuous and decreasing on $[0, 1]$

construction

$f_k(x) = x^{-1} \cdot n_k(x)$ is a continuous and strictly decreasing bijection from $[0, 1]$ to $[0, +\infty]$.

$$
\begin{array}{lcl}
n_k(c) & := & \sum_m \beta_m \phi_m^k(c) \\
n(c) & := & \sum_k n_k(c)
\end{array}
$$

existence of price

Define an endomap of $[0,1]^K$ as:

$$F_y(c) \quad = \quad f_y^{-1}(\sum_k n_k(c_k)) \quad = \quad f_y^{-1}(n(c))$$

- $F_y(c) \leq 1$ with equality if $c = \mathbf{1}$, $0 < F_y(\mathbf{0}) < 1$,
- F is a continuous and strictly increasing function
- the iterated sequence $F^n(\mathbf{0})$ is strictly increasing in all coordinates
- as it is bounded it converges to a fixed point $c^\star$
- $c^\star$ is a solution to the market balance such that $\sum c_k^\star = 1$

$$n(c^\star) = \sum_k n_k(c_k^\star) = f_y(c^\star) = 1/c^\star \cdot n_y(c^\star)$$

the market “procedure” (2)

One can think of $c' = F_y(c)$ as the new market price subsequent to a transaction; this assignment verifies that for all ys, $1/c'_y \cdot n_y(c') = n(c)$, where the lhs is what the market owes if y happens, and the rhs is an under-approximation of the total bet. Note that this does not suppose that the market knows what the ϕ^k_ms are; as one can define c'_y as the ratio between the total current bet on y and the total bet