Tutorial for week 8 (November 9–13)

Quiz tutorial

Tutorial structure: 25 minutes for the quiz, followed by 25 minutes for marking and discussing solutions.

1. Here is a small propositional Prolog program.

   godot :- waiting.
   waiting :- tick, waiting.
   tick.

   (a) Rewrite the program in standard logical notation.
   (b) Draw the search tree resulting from the initial query ?- godot.
   (c) What response does Prolog give to this query?
   (d) What response would a decision procedure for propositional logic give to the query godot?

2. Consider the following program operating on unary natural numbers, as seen in Programming lecture 3.

   even(z).
   odd(s(X)) :- even(X).
   even(s(X)) :- odd(X).

   (a) Write the program in logical notation, with each line written as a sentence with all quantifiers given explicitly.
   (b) Write the query below in logical notation, again with all quantifiers given explicitly.

      ?- odd(X), even(X).
(c) Consider three structures $S_1, S_2, S_3$ defined as follows. The sets $U_1, U_2, U_3$ are the corresponding universes (or domains) for the interpretations:

$$U_1 = \mathbb{N} \quad \text{i.e. } \{1, 2, 3, \ldots\}$$
$$U_2 = \{0, 1, 2\}$$
$$U_3 = \{0, 1, 2\}$$

The interpretation of the constant $z$, function symbol $s$ and predicate symbols $\text{even}$ and $\text{odd}$ are given as follows (here, for example, $\text{even}^{S_1}$ is the interpretation of the predicate even in $S_1$):

- $z^{S_1} = 0$
- $z^{S_2} = 0$
- $z^{S_3} = 0$
- $s^{S_1}(x) = x + 1$
- $s^{S_2}(x) = x + 1 \mod 3$
- $s^{S_3}(x) = x + 1 \mod 3$
- $\text{even}^{S_1}(x) = \text{x is even}$
- $\text{even}^{S_2}(x) = \text{x is even}$
- $\text{even}^{S_3}(x) = \text{is always true}$
- $\text{odd}^{S_1}(x) = \text{x is odd}$
- $\text{odd}^{S_2}(x) = \text{x is odd}$
- $\text{odd}^{S_3}(x) = \text{is always true}$

Which of these structures are models of the program?

(d) The query in part (b) is not a logical consequence of the program. Justify this statement.

(e) Is $\neg \exists X (\text{even}(X) \land \text{odd}(X))$ a logical consequence of the program?

(f) Is $\exists X \text{odd}(X)$ a logical consequence of the program?