Tutorial for week 8 (November 9–13)

Quiz tutorial

Tutorial structure: 25 minutes for the quiz, followed by 25 minutes for marking and discussing solutions.

1. Here is a small propositional Prolog program.

```
godot :- waiting.
waiting :- tick, waiting.
tick.
```

   (a) Rewrite the program in standard logical notation.

   (b) Draw the search tree resulting from the initial query `?- godot.`

   (c) What response does Prolog give to this query?

   (d) What response would a decision procedure for propositional logic give to the query `godot`?

2. Consider the following program operating on unary natural numbers, as seen in Programming lecture 3.

```
even(z).
odd(s(X)) :- even(X).
even(s(X)) :- odd(X).
```

   (a) Write the program in logical notation, with each line written as a sentence with all quantifiers given explicitly.

   (b) Write the query below in logical notation, again with all quantifiers given explicitly.

```
?- odd(X), even(X).
```

(c) Consider three structures S_1, S_2, S_3 defined as follows. The sets U_1, U_2, U_3 are the corresponding universes (or domains) for the interpretations:

$$U_1 = \mathbb{N} \qquad \text{i.e. } \{1, 2, 3, \ldots\}$$
$$U_2 = \{\ 0, 1, 2\ \}$$
$$U_3 = \{\ 0, 1, 2\ \}$$

The interpretation of the constant z, function symbol s and predicate symbols even and odd are given as follows (here, for example, even^{S_1} is the interpretation of the predicate even in S_1):

$z^{S_1} = 0$	$z^{S_2} = 0$	$z^{S_3} = 0$
$s^{S_1}(x) = x + 1$	$s^{S_2}(x) = x + 1 \text{ mod } 3$	$s^{S_3}(x) = x + 1 \text{ mod } 3$
$\text{even}^{S_1}(x) = x$ is even	$\text{even}^{S_2}(x) = x$ is even	$\text{even}^{S_3}(x)$ is always true
$\text{odd}^{S_1}(x) = x$ is odd	$\text{odd}^{S_2}(x) = x$ is odd	$\text{odd}^{S_3}(x)$ is always true

Which of these structures are models of the program?

(d) The query in part (b) is not a logical consequence of the program. Justify this statement.

(e) Is $\neg\exists X\ (even(X) \wedge \text{odd}(X))$ a logical consequence of the program?

(f) Is $\exists X\ \text{odd}(X)$ a logical consequence of the program?