Logic Programming: Search Strategies

Alan Smaill

Oct 19, 2015
Today

- Problem representation
- Search
  - Depth First
  - Iterative Deepening
  - Breadth First
- AND/OR (alternating/game tree) search
Search Problems

Many classical AI/CS problems can be formulated as search problems.

Examples:
- Graph searching
- Blocks world
- Missionaries and cannibals
- Planning (e.g. robotics)
Search Spaces

Given by:

- Set of states $s_1, s_2, \ldots$
- **Goal** predicate $\text{goal}(X)$
- **Step** predicate $s(X, Y)$ that says we can go from state $X$ to state $Y$
- A **start state** (or states)

- A **solution** is a path leading from the $S$ to a goal state $G$ satisfying $\text{goal}(G)$. 
Example: Blocks world

Take configuration of blocks as a list of three towers, each tower being a list of blocks in a tower from top to bottom.

\[
[[c, b, a], [], [c]]
\]
Example: Blocks world

Move a block from top of a tower to top of another tower:

```
[[b,a],[],[c]]
```
Example: Blocks world

Next move:

[[a], [b], [c]]
Example: Blocks world

Then —

[[],[a,b],[c]]
Prolog representation

- State is a list of stacks of blocks:
  \[
  [[a,b,c],[],[]]
  \]

- Transitions move a block from the top of one stack to the top of another:
  \[
  s([[A|As],Bs,Cs], [As,[A|Bs],Cs]).
  s([[A|As],Bs,Cs], [As,Bs,[A|Cs]]).
  \]
  ...

- Can specify particular goal position:
  \[
  goal([[],[],[a,b,c]]).
  \]
An abstract problem space

Think of the graph generated by these declarations.

\begin{align*}
s(a, b). \\
s(b, c). \\
s(c, a). \\
s(c, f(d)). \\
s(f(N), f(g(N))). \\
s(f(g(X)), X). \\
goal(d).
\end{align*}

In this case:

- the graph is infinite
- there is a loop near the top of the graph
abstract space ctd
problem 1: cycles

We can already see in the blocks world example and in the abstract search space that it is easy to follow actions around in cycles, and not find the goal, even if there is a path to the goal.

There are two main approaches to deal with this:
- remember where you’ve been; OR . . .
- work with depth bound
Solution 1: remember where you’ve been

% dfs( PathSoFar, CurrentNode, PathToGoal )

dfs_noloop(Path,Node,[Node|Path]) :-
    goal(Node).

dfs_noloop(Path,Node,Path1) :-
    s(Node, Node1),
    \+ member(Node1,Path),
    dfs_noloop([Node|Path], Node1, Path1).
Problem 2: Infinite State Space

Compare the graph from the abstract search space. Depth First Search has similar problems to Prolog proof search:

- We may miss solutions because state space is infinite;
- Even if state space is finite, may wind up finding “easy” solution only after a long exploration of pointless part of search space.
Solution 2: depth bounding

- Keep track of depth, stop if bound exceeded
  - Note: does not avoid loops (can do this too)

```prolog
dfs_bound(_,Node,[Node]) :-
goal(Node).

dfs_bound(N,Node,[Node|Path]) :-
  N > 0,
  s(Node,Node1),
  M is N-1,
  dfs_bound(M,Node1,Path)
```
Problem 3: what is a good bound?

- In general, we just don’t know in advance:
  - Too low? – Might miss solutions
  - Too high? – Might spend a long time searching pointlessly
Solution 3: iterative deepening

Use the following with some small start value for $N$

$$\text{dfs_id}(N, \text{Node}, \text{Path}) :-$$
$$\text{dfs_bound}(N, \text{Node}, \text{Path}) ;$$
$$M \text{ is } N+1,$$
$$\text{dfs_id}(M, \text{Node}, \text{Path}).$$

NB: if there is no solution, this will not terminate.
Breadth first search

Keep track of all possible solutions, try shortest ones first; do this by maintaining a “queue” of solutions

\[ \text{bfs}([\text{Node|Path}]\_\_], [\text{Node|Path}]) :\]
\[ \text{goal(Node)}. \]

\[ \text{bfs}([\text{Path|Paths}], S) :\]
\[ \text{extend(Path,NewPaths)}, \]
\[ \text{append(Paths,NewPaths,Paths1)}, \]
\[ \text{bfs(Paths1,S)}. \]

\[ \text{bfs_start(N,P) :} \text{fs}([\text{[N]}],P). \]
extending paths

```
extend([Node|Path], NewPaths) :-
    bagof([NewNode,Node|Path],
          (s(Node,NewNode),
           \+ (member(NewNode,[Node|Path]))),
          NewPaths),
    !.
%% if there are no next steps,
%% bagof will fail and we’ll fall through.

extend(_Path,[]).
```
Problem: speed

- Concatenating new paths to end of list is slow
- Avoid this using difference lists?
So far we've considered graph search problems
  - Just want to find some path from start to end
Other problems have more structure
  - e.g. 2-player games
AND/OR search is a useful abstraction
Example: Noughts and Crosses

MAX (x)

MIN (o)

MAX (x)

MIN (o)

TERMINAL

Utility

-1 0 +1
or(S, Nodes)
  ▸ S is an OR node with possible next states Nodes
  ▸ “Our move”
and(S, Nodes)
  ▸ S is an AND node with possible next states Nodes
  ▸ “Opponent moves”
goal(S)
  ▸ S is a “win” for us
Example: A simple game

\[
\begin{align*}
&\text{and}(a,[b,c]). \\
&\text{or}(b,[d,a]). \\
&\text{or}(c,[d,e]). \\
&\text{goal}(e).
\end{align*}
\]

What is the graph here?
Basic Idea

andor(Node) :- goal(Node).
andor(Node) :-
    or(Node,Nodes),
    member(Node1,Nodes),
    andor(Node1).
andor(Node) :-
    and(Node,Nodes),
    solveall(Nodes).

solveall(Nodes) :- ...
For each AND state, we need solutions for all possible next states
For each OR state, we just need one choice
A “solution” is thus a tree, or strategy

- Can adapt previous program to produce solution tree;
- Can also incorporate iterative deepening, loop avoidance, BFS.
- Heuristic measures of “good” positions leads to algorithms like MiniMax.
Noughts and crosses via minimax

See

http:

//www.emse.fr/~picard/cours/ai/minimax/

with acknowledgements to EMSE.

This provides alongside an implementation of minimax, instantiation to noughts and crosses (= tic-tac-toe), and a basic interface for playing the game.
Further Reading

- Bratko, Prolog Programming for Artificial Intelligence
  - ch. 8 (difference lists), ch. 11 (DFS/BFS)
  - also Ch. 12 (BestFS), 13 (AND/OR)