Logic Programming (Theory)

Assessed Coursework

This coursework is due at **3:00 pm, Monday 23rd November, 2015.**
You should hand in your solutions on paper to the ITO; they may be hand-written, formatted as you wish, or some combination of these.

1. (a) Consider the following Prolog program and query.

   \[\begin{array}{l}
d( f(X), f(Y) ) :- d( X, Y ). \\
d( f(X), Y ) :- d( X, Y ). \\
d( g(X), g(Y) ) :- d( X, Y ). \\
d( a, a ).
\end{array}\]

\[-d( f(g(a)), Z ), d( g(f(a)), Z ).\]

Rewrite the program and query in the language of first-order logic, writing all quantifiers explicitly.

   [6 marks]

(b) Draw the full search tree for the above program and query.

   [10 marks]

(c) Consider the three terms below:

   \[f( g(Y), h(Y) ), f( g(Y), h(Z) ), f( g(Y), Y )\]

For each pair of thee terms, say if the two terms are unifiable, and do one of the following:

i. if they are unifiable, say what the most general unifier is, and give another unifier which is not most general;
ii. if the two terms are not unifiable, then explain why not.

   [9 marks]
2. (a) Suppose that $f$ is a function from a power set of a set $X$ to itself:

$$f : \mathcal{P}(X) \rightarrow \mathcal{P}(X)$$

What does it mean to say:

i. $f$ is monotone?
ii. $Y$ is a fixed point of $f$?

[4 marks]

(b) Consider the following propositional logic program.

\[
\begin{align*}
  a & : = b, \ d. \\
  b & : = b. \\
  d & : = d. \\
  c & : = e. \\
  d. 
\end{align*}
\]

The meaning of the program is defined by the least fixed point of a function $f : \mathcal{P}(\{a, b, c, d, e\}) \rightarrow \mathcal{P}(\{a, b, c, d, e\})$. Explain what the function $f$ is, and calculate sufficiently many iterated applications of $f$ to the empty set to find the least fixed point of $f$.

[10 marks]

(c) How many models does the above program have? (i.e. how many interpretations of the atoms $\{a, b, c, d, e\}$ are there which make the program statements all true?)

[5 marks]

(d) Explain what is means by a decision procedure for definite clause propositional logic. Describe one possible decision procedure in detail.

[6 marks]