Logic Programming, Fall 2014
Tutorial 7: Herbrand Models and Completion Procedures

For discussion during Week 9 (Nov. 10 – Nov. 14)

1. We consider the least-fixed-point interpretation of a program in the context of predicate logic.

   Consider the following program (in Prolog notation).

```
flies(X) :- bird(X).
flies(X) :- bee(X).
bird(X) :- parrot(X).
bird(X) :- skua(X).
bee(X) :- bumbleBee(X).
parrot(polly).
skua(sam).
bumbleBee(bert).
penguin(pete).
```

   (a) What are the constants, function symbols and predicates?

   (b) What is the set of ground terms?

   (c) What is the set X of ground atomic formulas?

   (d) Define the monotone function $f\colon \mathcal{P}(X) \to \mathcal{P}(X)$ associated with the above program.

   (e) Compute iterations of f starting with the empty set $\{\,\}$ until the least fixed point is reached.

   (f) What is the minimal Herbrand model for the program?

2. Consider the following theory T (in logical notation).

$$\begin{aligned} \forall \mathtt{X}.\ \mathtt{poor}(\mathtt{X}) &\to \mathtt{happy}(X) \\ &\ \mathtt{poor}(\mathtt{karen}) \\ &\ \mathtt{rich}(\mathtt{keith}) \end{aligned}$$

   (a) Write out the theory, $CWA(T)$, that results from the theory T by imposing the *Closed World Assumption (CWA)*.

   (b) Use this example to illustrate the non-monotonicity of the CWA.

   (c) For this example, does the theory $CWA(T)$ correctly predict the Prolog behaviour of negation by failure on ground atomic formulas?

   (d) Write out the *Clark completion*, $Comp(T)$, of the theory T.

(e) Does the process of taking the Clark completion lead to a similar *non-monotonicity* as in (b)?

3. The CWA is applicable to general first-order theories. That is, it is not restricted to theories comprising of definite clauses. What is the result of applying the CWA to the theory consisting of the single formula below?

$$\texttt{happy}(\texttt{tweedledum}) \vee \texttt{happy}(\texttt{tweedledee})$$

What is the problem with the resulting theory?

4.* Given a program P in definite clause predicate logic, and a goal $G_1, \ldots, G_m$ (where $G_1, \ldots, G_m$ are atomic formulas), the problem of whether or not the logical consequence

$$P \models \exists \mathit{Vars}(G_1, \ldots, G_m).\ G_1 \wedge \cdots \wedge G_m$$

holds is undecidable. (This is explained in Lecture 5.) Show that this problem is decidable in the special case in which no function symbols appear in the program and query. That is, the program and query must be written using just predicate symbols and constants. (Hint: what special property does the set of all ground atomic formulas enjoy in this case?)