1. Here is a small propositional Prolog programs.

godot :- waiting.
waiting :- tick, waiting.
tick.

(a) Rewrite the program in standard logical notation.
(b) Draw the search tree resulting from the initial query godot.
(c) What response does Prolog make to the query godot?
(d) What response would a decision procedure for propositional logic make to the query godot?

2. Consider the following Prolog program operating on unary natural numbers (cf. slide 13, Programming Lecture 3).

even(z).
odd(s(X)) :- even(X).
even(s(X)) :- odd(X).

(a) Write this program in logical notation, with each line written as a sentence with all quantifiers explicitly given.
(b) Write the query below in logical notation, again with all quantification explicitly given.

?- odd(X), even(X).

(c) Consider three structures $S_1$, $S_2$ and $S_3$ defined as follows. The corresponding universes are $U_1$, $U_2$ and $U_3$ as specified below.

\[
\begin{align*}
U_1 &= \mathbb{N} \\
U_2 &= \{0, 1, 2\} \\
U_3 &= \{0, 1, 2\}
\end{align*}
\]

The interpretation of the constant $z$, function symbol $s$, and predicate symbols even and odd are defined by:

\[
\begin{align*}
z^{S_1} &= 0 \\
z^{S_2} &= 0 \\
z^{S_3} &= 0 \\
s^{S_1}(x) &= x + 1 \\
s^{S_2}(x) &= x + 1 \mod 3 \\
s^{S_3}(x) &= x + 1 \mod 3 \\
\text{even}^{S_1}(x) &\iff x \text{ is even} \\
\text{even}^{S_2}(x) &\iff x \text{ is even} \\
\text{even}^{S_3}(x) &\text{is always true} \\
\text{odd}^{S_1}(x) &\iff x \text{ is odd} \\
\text{odd}^{S_2}(x) &\iff x \text{ is odd} \\
\text{odd}^{S_3}(x) &\text{is always true}
\end{align*}
\]

Which of these structures are models of the program?
(d) The query in part (b) is not a logical consequence of the program. Justify this statement!

(e) Is \( \neg \exists X. \text{even}(X) \land \text{odd}(X) \) a logical consequence of the program?

(f) Is \( \exists X. \text{odd}(X) \) a logical consequence of the program?