Logic Programming
Tutorial 3: Logical Consequence

Week 5 QUIZ tutorial (Oct. 13–17)

Tutorial structure: 25 minutes for the quiz, followed by 25 minutes for marking and discussing solutions.

1. Here is a small propositional Prolog programs.

```
godot :- waiting.
waiting :- tick, waiting.
tick.
```

   (a) Rewrite the program in standard logical notation.

   (b) Draw the search tree resulting from the initial query `godot`.

   (c) What response does Prolog make to the query `godot`?

   (d) What response would a decision procedure for propositional logic make to the query `godot`?

2. Consider the following Prolog program operating on unary natural numbers (cf. slide 13, Programming Lecture 3).

```
even(z).
odd(s(X)) :- even(X).
even(s(X)) :- odd(X).
```

   (a) Write this program in logical notation, with each line written as a sentence with all quantifiers explicitly given.

   (b) Write the query below in logical notation, again with all quantification explicitly given.

```
?- odd(X), even(X).
```

   (c) Consider three structures $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ defined as follows. The corresponding universes are U_1, U_2 and U_3 as specified below.

$$\begin{aligned} U_1 &= \mathbb{N} && \text{(i.e., } \{0,1,2,3,\dots\}\text{)} \\ U_2 &= \{0,1,2\} \\ U_3 &= \{0,1,2\} \end{aligned}$$

   The interpretation of the constant `z`, function symbol `s`, and predicate symbols `even` and `odd` are defined by:

$\mathtt{z}^{\mathcal{S}_1} = 0$	$\mathtt{z}^{\mathcal{S}_2} = 0$	$\mathtt{z}^{\mathcal{S}_3} = 0$
$\mathtt{s}^{\mathcal{S}_1}(x) = x+1$	$\mathtt{s}^{\mathcal{S}_2}(x) = x+1 \bmod 3$	$\mathtt{s}^{\mathcal{S}_3}(x) = x+1 \bmod 3$
$\mathtt{even}^{\mathcal{S}_1}(x) \Leftrightarrow x$ is even	$\mathtt{even}^{\mathcal{S}_2}(x) \Leftrightarrow x$ is even	$\mathtt{even}^{\mathcal{S}_3}(x)$ is always true
$\mathtt{odd}^{\mathcal{S}_1}(x) \Leftrightarrow x$ is odd	$\mathtt{odd}^{\mathcal{S}_2}(x) \Leftrightarrow x$ is odd	$\mathtt{odd}^{\mathcal{S}_3}(x)$ is always true

   Which of these structures are models of the program?

(d) The query in part (b) is not a logical consequence of the program. Justify this statement!

(e) Is $\neg\,\exists \texttt{X}.\,\texttt{even(X)} \wedge \texttt{odd(X)}$ a logical consequence of the program?

(f) Is $\exists \texttt{X}.\,\texttt{odd(X)}$ a logical consequence of the program?